Dynamical Behavior of a Stochastic Nutrient-plankton Food Chain Model with Lévy Jumps*

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Abstract In aquatic ecosystem, plankton populations are easily affected by environmental fluctuations due to the unpredictability of many physical factors. To better understand how environmental fluctuations influence plankton populations, in this paper, we propose and investigate a stochastic nutrient-plankton food chain model with Lévy jumps. Firstly, by constructing a suitable Lyapunov function, we prove that the stochastic model has a unique global positive solution for any given positive initial value. Then, we establish sufficient conditions for the persistence and extinction of plankton. Finally, we provide some numerical simulations to illustrate the analytical results.

Keywords Stochastic nutrient-plankton model, persistence and extinction, Lévy jumps

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1. Introduction

Plankton is the basis of the marine food webs and supports various marine and terrestrial organisms, so it plays a key role in marine atmospheric dynamics [1]. In order to deeply understand the interaction between plankton and nutrient, many mathematical models have been developed in the past decades [2–6]. The nutrient-phytoplankton-zooplankton model incorporates one of the simplest sets of dynamics that usefully describe oceanic plankton dynamics [7]. For example, the author of [8] proposed the following nutrient-plankton food chain model:

$$\begin{cases} \dfrac{\mathrm{d}N(t)}{\mathrm{d}t} = D(N^0 - N) - aP(t)N(t) + \mu_3 P(t) + \mu_4 Z(t), \\ \dfrac{\mathrm{d}P(t)}{\mathrm{d}t} = aP(t)N(t) - bP(t)Z(t) - (\mu_1 + D_1)P(t), \\ \dfrac{\mathrm{d}Z(t)}{\mathrm{d}t} = bP(t)Z(t) - (\mu_2 + D_2)Z(t), \end{cases} \tag{1.1}$$

where $N(t)$, $P(t)$ and $Z(t)$ denote the concentrations of nutrient, phytoplankton and zooplankton at time t, respectively. The parameter a is the maximal nutrient uptake rate of phytoplankton, b is the maximal nutrient uptake rate of zooplankton,

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N^0 is the input concentration of nutrient, D is the washout rate of nutrient, D_1 is the washout rate of phytoplankton, D_2 is the washout rate of zooplankton, μ_1 is the death rate of phytoplankton, μ_2 is the death rate of zooplankton, μ_3 is the nutrient recycling rate from dead phytoplankton, and μ_4 is the nutrient recycling rate from dead zooplankton. From a biological perspective, we assume that $\mu_3 \leq \mu_1$ and $\mu_4 \leq \mu_2$. All parameters are positive.

It is well recognized that randomness can be seen everywhere in nature [9]. Of course, ecological populations are no exception. As revealed by May [10], biological parameters such as the birth rate, death rate and competition coefficient will fluctuate more or less randomly due to the randomness of the environment. For example, Zhao et al. [11] studied a stochastic phytoplankton allelopathy model under regime switching, revealing that the allelopathy effect plays a vital role in the existence of ergodic stationary distribution. Sarkar and Chattopadhayay [12] introduced environmental fluctuations into a phytoplankton-zooplankton system and concluded that the control of the intensity of noise is a key factor for the termination of planktonic blooms. Majumder et al. [13] presented a minimal model of phytoplankton-zooplankton interaction and compared its dynamics with the stochastic version extended by two different stochastic perturbation approaches, revealing that the persistence of species largely depends on the environmental noise. Imhof and Walcher [14] proposed a stochastic chemostat model for single substrate, showing that the white noise may make the microorganism extinct. Based on model (1.1), Yu et al. [15] proposed the following stochastic nutrient-plankton food chain model:

$$\begin{cases} \mathrm{d}N(t) = [D(N^0 - N) - aP(t)N(t) + \mu_3 P(t) + \mu_4 Z(t)]\mathrm{d}t \\ \qquad\quad + \sigma_1 N(t)\mathrm{d}B_1(t), \\ \mathrm{d}P(t) = [aP(t)N(t) - bP(t)Z(t) - (\mu_1 + D_1)P(t)]\mathrm{d}t \\ \qquad\quad + \sigma_2 P(t)\mathrm{d}B_2(t), \\ \mathrm{d}Z(t) = [bP(t)Z(t) - (\mu_2 + D_2)Z(t)]\mathrm{d}t + \sigma_3 z(t)\mathrm{d}B_3(t), \end{cases} \qquad (1.2)$$

where $B_i(t)(i = 1, 2, 3)$ represents the Brownian motion defined in the complete probability space $(\Omega, \mathbb{F}, \mathbb{P})$, and $\sigma_i(i = 1, 2, 3)$ stands for the intensity of white noise.

Meanwhile, the food chain may suffer sudden and serious environmental shocks, such as floods and toxic pollutants, which cannot be accurately described by model (1.2). To improve the situation, scholars turn to use the non-Gaussian Lévy noise to simulate these discontinuous abrupt environmental shocks in nature (see e.g. [16–20]). For example, Zeng et al. [16] considered a stochastic three species food chain model with general Lévy jumps. The criterion on the global stability in the mean with probability one for each species is established. The results showed that the dynamics of the model can be significantly changed by Lévy jumps. Gao et al. [17] investigated a Lotka-Volterra food chain chemostat model in random environment. They proved the existence and uniqueness of the global positive solution and further obtained the conditions for the extinction and persistence in the mean of microorganisms. Lu and Ding [18] considered a stochastic competitive model with infinite delay and general Lévy jumps. Sufficient conditions for stability in time average are established as well as permanence in time average and extinction. Zhu et al. [21] investigated a stochastic nonautonomous Gompertz model with Lévy jumps. The existences of a global positive solution and an explicit solution have

been derived.

Motivated by aforementioned, we present a stochastic nutrient-plankton food chain model driven by Lévy jumps and then investigate the persistence and extinction of the plankton. To do this, let $p(t, \omega)$ denote a stationary $\mathcal{F}_t$−adapted Poisson point process. N_1 is the Poisson counting measure generated by $p(t, \omega)$. λ is the intensity measure of N_1, which is defined on a finite measurable subset $\mathbb{Y}$ of $(0, \infty)$ with $\lambda(\mathbb{Y}) = 1$. $\widetilde{N}$ is the compensated random measure defined by $\widetilde{N}(dt, du) = N_1(dt, du) - \lambda(du)dt$. Note that the energy obtained by zooplankton and phytoplankton is consumed, and cannot be completely used for their own growth. Then model (1.2) can be modified to the following stochastic nutrient-plankton food chain model driven by Lévy jumps:

$$\begin{cases} dN(t) = [D(N^0 - N) - aP(t)N(t) + \mu_3 P(t) + \mu_4 Z(t)]dt \\ \qquad\quad + \sigma_1 N(t)dB_1(t) + \int_{\mathbb{Y}} \gamma_1(u)N(t^-)\widetilde{N}(dt, du), \\ dP(t) = [aa_1 P(t)N(t) - bP(t)Z(t) - (\mu_1 + D_1)P(t)]dt \\ \qquad\quad + \sigma_2 P(t)dB_2(t) + \int_{\mathbb{Y}} \gamma_2(u)P(t^-)\widetilde{N}(dt, du), \\ dZ(t) = [bb_1 P(t)Z(t) - (\mu_2 + D_2)Z(t)]dt + \sigma_3 z(t)dB_3(t) \\ \qquad\quad + \int_{\mathbb{Y}} \gamma_3(u)Z(t^-)\widetilde{N}(dt, du), \end{cases} \tag{1.3}$$

where a_1 stands for the fraction of phytoplankton nutrient conversion and b_1 is the fraction of zooplankton nutrient conversion, and γ_i denotes the intensities of Lévy jumps with $i = 1, 2, 3$. $N(t^-)$, $P(t^-)$ and $Z(t^-)$ denote the left limits of $N(t)$, $P(t)$ and $Z(t)$, respectively.

The organization of this paper is as follows. In Section 2, we first present some preliminary results. In Section 3, we perform the existence and uniqueness of the global positive solution for model (1.3). Then, some sufficient conditions for the persistence and extinction of model (1.3) are presented in Section 4. In Section 5, we give numerical simulations to verify the obtained theoretical results. Finally, a brief discussion is presented in Section 6.

2. Preliminaries

First, let us recall some basic characteristics of Lévy jumps [22]. Define L_t to be a $\mathbb{R}$-valued stochastic process on the probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathcal{P})$, where $t \in [0, \widetilde{T}]$ and $0 < \widetilde{T} < \infty$. It is well known that a Lévy process includes a given family of infinitely divisible distributions and has the following three properties:

(a) $L_0 = 0$, almost surely (a.s.). Sample paths are right continuous with left-limit a.s.

(b) Let $L_{t^-} = \lim_{\varepsilon \to 0} L_{t-\varepsilon}$. Then $L_t = L_{t^-}$ holds a.s. for any $t > 0$.

(c) It has independent increments, i.e., for any $0 \le t_0 < t_1 < \cdots \le \widetilde{T}$ and $i \in \mathbb{N}$, the random variables $L_{t_i} - L_{t_{i-1}}$ are independent.

Any Lévy process consists of three components: a linear drift, a Brownian motion $B_i(t)$ and a superposition of independent (centered) Poisson processes with

different jump sizes $\lambda(\mathrm{d}u)$, which represents the rate of arrival (intensity) of the Poisson process with the jump of size u [23].

For the sake of convenience and simplicity, we introduce the following notations.

$$
\begin{aligned}
&\mathbb{R}^n_+ = \{x \in \mathbb{R}^n | x_j > 0, j = 1, 2, \cdots, n\}, \ \mathbb{N} = \{1, 2, \ldots\}, \\
&\langle f(t) \rangle = \frac{1}{t} \int_0^t f(s)\mathrm{d}s, \ f^* = \limsup_{t \to \infty} f(t), \ f_* = \liminf_{t \to \infty} f(t), \\
&\eta_i = \int_{\mathbb{Y}} [\gamma_i(u) - \ln(1 + \gamma_i(u))]\lambda(\mathrm{d}u), \ \beta_i = \frac{1}{2}\sigma_i^2 + \eta_i, \\
&M_i(t) = \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_i(u))\widetilde{\mathrm{N}}(\mathrm{d}s, \mathrm{d}u), i = 1, 2, 3.
\end{aligned}
\tag{2.1}
$$

It is worth mentioning that η_i denotes the accumulation effect of Lévy jumps (sudden environmental shocks, described by a surperposition of independent Poisson process) in a measurable subset $\mathbb{Y}$ [23].

To facilitate the description, we first introduce the following two lemmas.

Lemma 2.1. *For model* (1.3)*, if* $\lim_{t\to\infty} P(t) = 0$ *a.s., then* $\lim_{t\to\infty} Z(t) = 0$ *a.s.*

The proof of this lemma is simple, so it is omitted here. For the detailed proof process, please refer to [26].

Lemma 2.2 ([27])**.** *Suppose* $x(t) \in C[\Omega \times [0, +\infty), \mathbb{R}_+]$.

(i) *If there exist two positive constants* λ_0 *and* T*, such that*

$$
\ln x(t) \leq \lambda t - \lambda_0 \int_0^t x(s)ds + \alpha \sum_{i=1}^3 B_i(t) + \sum_{i=1}^3 \delta_i M_i(t) \ a.s.
$$

for all $t \geq T$*, where* α*,* δ_i *are constants, then*

$$
\begin{cases}
\langle x \rangle^* \leq \frac{\lambda}{\lambda_0} \ a.s. \ \ if \lambda \geq 0; \\
\lim_{t\to\infty} x(t) = 0 \ a.s. \ \ if \lambda < 0.
\end{cases}
$$

(ii) *If there exist three positive constants* λ_0*,* λ *and* T*, such that*

$$
\ln x(t) \geq \lambda t - \lambda_0 \int_0^t x(s)ds + \alpha \sum_{i=1}^3 B_i(t) + \sum_{i=1}^3 \delta_i M_i(t) \ a.s.
$$

for all $t \geq T$*, then* $\langle x \rangle_* \geq \frac{\lambda}{\lambda_0}$ *a.s.*

3. Existence and uniqueness of the global positive solution

In this section, we study the existence and uniqueness of the global positive solution, which is prerequisite for analyzing the long-term behavior of model (1.3).

Theorem 3.1. *For any initial value* $(N(0), P(0), Z(0)) \in \mathbb{R}^3_+$*, there exists a unique solution* $(N(t), P(t), Z(t))$ *of system* (1.3) *on* $t \geq 0$ *and the solution will remain in* $\mathbb{R}^3_+$ *with probability one.*

Proof. Since the coefficients of system (1.3) satisfy the local Lipschitz condition, then for any initial value $(N(0), P(0), Z(0)) \in \mathbb{R}_+^3$, there is a unique local solution $(N(t), P(t), Z(t))$ on $t \in [0, \tau_e)$, where τ_e denotes the explosion time. To show this solution is global, we only need to prove $\tau_e = \infty$ a.s. To this end, let $k_0 \geq 1$ be sufficiently large such that $N(0)$, $P(0)$ and $Z(0)$ all lie within the interval $[\frac{1}{k_0}, k_0]$. For each interval $k \geq k_0$, define the stopping time as

$$\tau_k = \inf \left\{ t \in [0, \tau_e) : \min \left\{ N(t), P(t), Z(t) \right\} \leq \frac{1}{k} \ or \ \max \left\{ N(t), P(t), Z(t) \right\} \geq k \right\}.$$

Obviously, τ_k is increasing as $k \to \infty$. Let $\tau_\infty = \lim_{k \to \infty} \tau_k$, hence $\tau_\infty \leq \tau_e$ a.s. If $\tau_\infty = \infty$ a.s. is true, then $\tau_e = \infty$ a.s. and $(N(t), P(t), Z(t)) \in \mathbb{R}_+^3$ a.s. for all $t \geq 0$. That is to say, to complete the proof, we only need to prove $\tau_\infty = \infty$ a.s. If this statement is false, then there is a pair of constants $T > 0$ and $\varepsilon \in (0, 1)$ such that

$$P\{\tau_\infty \leq T\} > \varepsilon.$$

Hence there is an integer $k_1 \geq k_0$ such that

$$P\{\tau_k \leq T\} > \varepsilon. \tag{3.1}$$

Define a C^2-function $\tilde{V} : \mathbb{R}_+^3 \to \mathbb{R}_+$ by

$$\widetilde{V}(N, P, Z) = (N - c - c \ln \frac{N}{c}) + (P - 1 - \ln P) + (Z - 1 - \ln Z), \tag{3.2}$$

where c is a positive constant to be determined later. The nonnegativity of the function (3.2) can be seen from

$$u - 1 - \ln u \geq 0.$$

Let $k \geq k_1$ and $T > 0$ be arbitrary. For any $0 \leq t \leq \tau_k \wedge T = \min\{\tau_k, T\}$, applying the generalized Itô's formula to $\widetilde{V}$ leads to

$$\begin{aligned} \mathrm{d}\widetilde{V}(N, P, Z) =& L\widetilde{V}(N, P, Z)\mathrm{d}t + \sigma_1(N - c)\mathrm{d}B_1(t) + \sigma_2(P - 1)\mathrm{d}B_2(t) \\ &+ \sigma_3(Z - 1)\mathrm{d}B_3(t) + \int_{\mathbb{Y}} [\gamma_1(u)N(t) - \ln(1 + \gamma_1(u))]\widetilde{N}(\mathrm{d}t, \mathrm{d}u) \\ &+ \int_{\mathbb{Y}} [\gamma_2(u)P(t) - \ln(1 + \gamma_2(u))]\widetilde{N}(\mathrm{d}t, \mathrm{d}u) \\ &+ \int_{\mathbb{Y}} [\gamma_3(u)Z(t) - \ln(1 + \gamma_3(u))]\widetilde{N}(\mathrm{d}t, \mathrm{d}u), \end{aligned}$$

where

$$\begin{aligned} L\widetilde{V}(N, P, Z) =& (1 - \frac{c}{N})[D(N^0 - N) - aP(t)N(t) + \mu_3 P(t) + \mu_4 Z(t)] \\ &+ (1 - \frac{1}{P})[aa_1 P(t)N(t) - bP(t)Z(t) - (\mu_1 + D_1)P(t))] \\ &+ (1 - \frac{1}{Z})[bb_1 P(t)Z(t) - (\mu_2 + D_2)Z(t)] \\ &+ \frac{1}{2}(c\sigma_1^2 + \sigma_2^2 + \sigma_3^2) + \int_Y [c\gamma_1 - c\ln(1 + \gamma_1)]\lambda(\mathrm{d}u) \end{aligned}$$

$$+ \int_Y [\gamma_2 - \ln(1 + \gamma_2)]\lambda(\mathrm{d}u) + \int_Y [\gamma_3 - \ln(1 + \gamma_3)]\lambda(\mathrm{d}u)$$

$$\leq [DN^0 + cD + \mu_1 + D_1 + \mu_2 + D_2 + \frac{1}{2}(c\sigma_1^2 + \sigma_2^2 + \sigma_3^2)]$$

$$+ (-D - aa_1)N(t) + [\mu_3 + ac - (\mu_1 + D_1) - bb_1]P(t)$$

$$+ [\mu_4 + b - (\mu_2 + D_2)]Z(t) + (aa_1 - a)\,P(t)N(t)$$

$$+ (bb_1 - b)\,P(t)Z(t) + c\eta_1 + \eta_2 + \eta_3$$

$$\leq [DN^0 + cD + \mu_1 + D_1 + \mu_2 + D_2 + \frac{1}{2}(c\sigma_1^2 + \sigma_2^2 + \sigma_3^2)]$$

$$+ [\mu_3 + ac - (\mu_1 + D_1) - bb_1]P(t) + c\eta_1 + \eta_2 + \eta_3.$$

Choose $c = \frac{\mu_1 + D_1 + bb_1 - \mu_3}{a}$ such that $\mu_3 + ac - (\mu_1 + D_1) - bb_1 = 0$. Then we get

$$L\widetilde{V}(N, P, Z) \leq [DN^0 + cD + \mu_1 + D_1 + \mu_2 + D_2 + \frac{1}{2}(c\sigma_1^2 + \sigma_2^2 + \sigma_3^2)]$$

$$+ c\eta_1 + \eta_2 + \eta_3 := K,$$

where K is a positive constant. Hence,

$$\begin{aligned}
\mathrm{d}\widetilde{V}(N, P, Z) \leq & K\mathrm{d}t + \sigma_1(N - c)\mathrm{d}B_1(t) + \sigma_2(P - 1)\mathrm{d}B_2(t) \\
& + \sigma_3(Z - 1)\mathrm{d}B_3(t) + \int_{\mathbb{Y}} [\gamma_1 N(t) - \ln(1 + \gamma_1)]\widetilde{\mathrm{N}}(\mathrm{d}t, \mathrm{d}u) \\
& + \int_{\mathbb{Y}} [\gamma_2 P(t) - \ln(1 + \gamma_2)]\widetilde{\mathrm{N}}(\mathrm{d}t, \mathrm{d}u) \\
& + \int_{\mathbb{Y}} [\gamma_3 Z(t) - \ln(1 + \gamma_3)]\widetilde{\mathrm{N}}(\mathrm{d}t, \mathrm{d}u).
\end{aligned} \tag{3.3}$$

Integrating both sides of (3.3) from 0 to $\tau_k \wedge T = \min\{\tau_k, T\}$ yields

$$\begin{aligned}
\int_0^{\tau_k \wedge T} \mathrm{d}\widetilde{V}(N, P, Z) \leq & \int_0^{\tau_k \wedge T} K\mathrm{d}t + \sigma_1 \int_0^{\tau_k \wedge T} (N(s) - c)\mathrm{d}B_1(s) \\
& + \sigma_2 \int_0^{\tau_k \wedge T} (P(s) - 1)\mathrm{d}B_2(s) + \sigma_3 \int_0^{\tau_k \wedge T} (Z(s) - 1)\mathrm{d}B_3(s) \\
& + \int_0^{\tau_k \wedge T} \int_{\mathbb{Y}} [\gamma_1 N(s) - c\ln(1 + \gamma_1)]\widetilde{\mathrm{N}}(\mathrm{d}s, \mathrm{d}u) \\
& + \int_0^{\tau_k \wedge T} \int_{\mathbb{Y}} [\gamma_2 P(s) - \ln(1 + \gamma_2)]\widetilde{\mathrm{N}}(\mathrm{d}s, \mathrm{d}u) \\
& + \int_0^{\tau_k \wedge T} \int_{\mathbb{Y}} [\gamma_3 Z(s) - \ln(1 + \gamma_3)]\widetilde{\mathrm{N}}(\mathrm{d}s, \mathrm{d}u).
\end{aligned}$$

Taking the expectation on both sides leads to

$$\begin{aligned}
E\widetilde{V}(N(\tau_k \wedge T), P(\tau_k \wedge T), Z(\tau_k \wedge T)) \leq & \widetilde{V}(N(0), P(0), Z(0)) + KE(\tau_k \wedge T) \\
& \leq \widetilde{V}(N(0), P(0), Z(0)) + KT.
\end{aligned} \tag{3.4}$$

Set $\Omega_k = \{\tau_k \leq T\}$ for $k \geq k_1$. By (3.1), we have $P(\Omega_k) \geq \varepsilon$. Note that for every $\omega \in \Omega_k$, there are $N(\tau_k, \omega)$ or $P(\tau_k, \omega)$ or $Z(\tau_k, \omega)$ which equals either k or $\frac{1}{k}$, so

$\widetilde{V}(N(\tau_k, \omega), P(\tau_k, \omega), Z(\tau_k, \omega))$ is no less than either

$$(k - c - c \ln \frac{k}{c}) \wedge (k - 1 - \ln k) \text{ or } (\frac{1}{k} - c + c \ln(ck)) \wedge (\frac{1}{k} - 1 + \ln k).$$

Hence

$$\widetilde{V}(N(\tau_k, \omega), P(\tau_k, \omega), Z(\tau_k, \omega) \geq$$
$$(k - c - c \ln \frac{k}{c}) \wedge (k - 1 - \ln k) \wedge (\frac{1}{k} - c + c \ln(ck)) \wedge (\frac{1}{k} - 1 + \ln k).$$

In view of (3.4), we have

$$\widetilde{V}(N(0), P(0), Z(0)) + KT \geq E[1_{\Omega_{k(\omega)}} \widetilde{V}(N(\tau_k, \omega), P(\tau_k, \omega), Z(\tau_k, \omega)]$$
$$\geq \varepsilon[(k - c - c \ln \frac{k}{c}) \wedge (k - 1 - \ln k) \wedge (\frac{1}{k} - c + c \ln(ck)) \wedge (\frac{1}{k} - 1 + \ln k)],$$

where 1_{Ω_k} denotes the indicator function of Ω_k. Letting $k \to \infty$ leads to the contradiction

$$\infty > \widetilde{V}(N(0), P(0), Z(0)) + KT = \infty.$$

Thus we must have $\tau_\infty = \infty$ a.s. This completes the proof. $\qquad\qquad\square$

4. The persistence and extinction of model (1.3)

In this section, we mainly perform the persistence and extinction of the phytoplankton and zooplankton for the stochastic model (1.3). For the convenience of discussion in the rest of this paper, we introduce

$$R_0^* = a_1 a N^0 - (\mu_1 + D_1 + \beta_2), \ R_2^* = \frac{a_1 b_1 b D N^0}{\mu_1 + D_1 - a_1 \mu_3} - (\mu_2 + D_2 + \beta_3),$$

$$R_1^* = D b b_1 (a a_1 N^0 - \mu_1 - D_1 - \beta_2) - a(\mu_2 + D_2 + \beta_3)(\mu_1 + D_1 - a_1 \mu_3),$$

$$Q_0 = \frac{R_2^*(\mu_1 + D_1 - a_1 \mu_3)}{b(\mu_2 + D_2 - a_1 b_1 \mu_4)}, \ Q_1 = R_0^* + \left[\frac{a a_1}{D} (\mu_4 - \frac{\mu_2 + D_2}{a_1 b_1}) - b \right] Q_0, \qquad (4.1)$$

$$Q_2 = -(\mu_2 + D_2 + \beta_3) + \frac{b b_1 D Q_1}{a(\mu_1 + D_1 - a_1 \mu_3)}.$$

The main objective of this paper is to perform the dynamical analysis of stochastic model (1.3). So we firstly discuss the extinct conditions.

Theorem 4.1. *For model* (1.3), *we have*

(i) *if* $R_0^* < 0$, *then*

$$\lim_{t \to \infty} P(t) = 0 \ \ and \ \ \lim_{t \to \infty} Z(t) = 0 \ a.s.,$$

which means that both phytoplankton and zooplankton will become extinct.

(ii) *if* $R_0^* \geq 0$ *and* $R_1^* < 0$, *then*

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t P(s) ds = \frac{D R_0^*}{a(\mu_1 + D_1 - a_1 \mu_3)} \ \ and \ \ \lim_{t \to \infty} Z(t) = 0 \ a.s.,$$

which means that phytoplankton is persistent and zooplankton tends to become extinct.

Proof. Adapting the methods from [24] and [25], for model (1.3), we have the following results:

$$
\begin{aligned}
&\lim_{t\to\infty}\sup[N(t)+P(t)+Z(t)]<\infty,\quad \lim_{t\to\infty}\frac{1}{t}\int_0^t \sigma_1 N(s)\mathrm{d}B_1(s)=0,\\
&\lim_{t\to\infty}\frac{1}{t}\int_0^t \sigma_2 P(s)\mathrm{d}B_2(s)=0,\quad \lim_{t\to\infty}\frac{1}{t}\int_0^t \sigma_3 Z(s)\mathrm{d}B_3(s)=0,\\
&\lim_{t\to\infty}\frac{\int_0^t\int_{\mathbb{Y}}\gamma_1(u)N(s)\widetilde{\mathrm{N}}(\mathrm{d}s,\mathrm{d}u)}{t}=0,\quad \lim_{t\to\infty}\frac{\int_0^t\int_{\mathbb{Y}}\gamma_2(u)P(s)\widetilde{\mathrm{N}}(\mathrm{d}s,\mathrm{d}u)}{t}=0,\\
&\lim_{t\to\infty}\frac{\int_0^t\int_{\mathbb{Y}}\gamma_3(u)Z(s)\widetilde{\mathrm{N}}(\mathrm{d}s,\mathrm{d}u)}{t}=0.
\end{aligned}
\tag{4.2}
$$

Making use of Itô's formula with jumps [28] to model (1.3), we have

$$
\begin{aligned}
\mathrm{d}\ln P(t)=&\left\{aa_1 N(t)-bZ(t)-(\mu_1+D_1)-\frac{\sigma_2^2}{2}+\int_{\mathbb{Y}}[\ln(1+\gamma_2(u))-\gamma_2(u)]\lambda du\right\}\mathrm{d}t\\
&+\sigma_2\mathrm{d}B_2(t)+\int_{\mathbb{Y}}\ln(1+\gamma_2(u))\widetilde{\mathrm{N}}(\mathrm{d}t,\mathrm{d}u).
\end{aligned}
$$

Integrating both sides from 0 to t and divided by t, yields,

$$
\begin{aligned}
\frac{1}{t}\ln\frac{P(t)}{P(0)}=&-(\mu_1+D_1+\beta_2)+\frac{1}{t}\int_0^t aa_1 N(s)\mathrm{d}s-\frac{1}{t}\int_0^t bZ(s)\mathrm{d}s\\
&+\frac{1}{t}\sigma_2 B_2(t)+\frac{1}{t}M_2(t).
\end{aligned}
\tag{4.3}
$$

Similary, we can get

$$
\frac{1}{t}\ln\frac{Z(t)}{Z(0)}=-(\mu_2+D_2+\beta_3)+\frac{1}{t}\int_0^t bb_1 P(s)\mathrm{d}s+\frac{1}{t}\sigma_3 B_3(t)+\frac{1}{t}M_3(t).
\tag{4.4}
$$

Set $U(N,P,Z)=N(t)+\frac{1}{a_1}P(t)+\frac{1}{a_1 b_1}Z(t)$. Then,

$$
\begin{aligned}
\mathrm{d}U(N,P,Z)=&\mathrm{d}N(t)+\frac{1}{a_1}\mathrm{d}P(t)+\frac{1}{a_1 b_1}\mathrm{d}Z(t)\\
=&\left\{(DN^0-DN)+\left(\mu_3-\frac{\mu_1+D_1}{a_1}\right)P(t)+\left(\mu_4-\frac{\mu_2+D_2}{a_1 b_1}\right)Z(t)\right\}\mathrm{d}t\\
&+\sigma_1 N(t)\mathrm{d}B_1(t)+\frac{\sigma_2}{a_1}P(t)\mathrm{d}B_2(t)+\frac{\sigma_3}{a_1 b_1}Z(t)\mathrm{d}B_3(t)\\
&+\int_{\mathbb{Y}}\gamma_1(u)N(t)\widetilde{\mathrm{N}}(\mathrm{d}t,\mathrm{d}u)+\frac{1}{a_1}\int_{\mathbb{Y}}\gamma_2(u)P(t)\widetilde{\mathrm{N}}(\mathrm{d}t,\mathrm{d}u)\\
&+\frac{1}{a_1 b_1}\int_{\mathbb{Y}}\gamma_3(u)Z(t)\widetilde{\mathrm{N}}(\mathrm{d}t,\mathrm{d}u).
\end{aligned}
$$

In virtue of the above equation, we have

$$
\begin{aligned}
D\frac{1}{t}\int_0^t N(s)\mathrm{d}s=&DN^0+\frac{1}{t}\int_0^t\left(\mu_3-\frac{\mu_1+D_1}{a_1}\right)P(s)\mathrm{d}s\\
&+\frac{1}{t}\int_0^t\left(\mu_4-\frac{\mu_2+D_2}{a_1 b_1}\right)Z(s)\mathrm{d}s+\frac{\varphi_1(t)}{t},
\end{aligned}
\tag{4.5}
$$

where

$$\varphi_1(t) = U(0) - U(N, P, Z) + \int_0^t \sigma_1 N(s)\mathrm{d}B_1(s) + \frac{\sigma_2}{a_1}\int_0^t P(s)\mathrm{d}B_2(s)$$

$$+ \frac{\sigma_3}{a_1 b_1}\int_0^t Z(s)\mathrm{d}B_3(s) + \int_0^t \int_{\mathbb{Y}} \gamma_1(u)N(s)\widetilde{N}(\mathrm{d}s, \mathrm{d}u)$$

$$+ \frac{1}{a_1}\int_0^t \int_{\mathbb{Y}} \gamma_2(u)P(s)\widetilde{N}(\mathrm{d}s, \mathrm{d}u) + \frac{1}{a_1 b_1}\int_0^t \int_{\mathbb{Y}} \gamma_3(u)Z(s)\widetilde{N}(\mathrm{d}s, \mathrm{d}u).$$

Due to (4.2), we get $\lim_{t\to\infty}\frac{\varphi_1(t)}{t} = 0$ a.s. Let us first prove (i). Substituting (4.5) into (4.3) leads to

$$\frac{1}{t}\ln\frac{P(t)}{P(0)} = aa_1 N^0 - (\mu_1 + D_1 + \beta_2) + \frac{aa_1}{D}[\frac{1}{t}\int_0^t \left(\mu_3 - \frac{\mu_1 + D_1}{a_1}\right)P(s)\mathrm{d}s$$

$$+ \frac{1}{t}\int_0^t \left(\mu_4 - \frac{\mu_2 + D_2}{a_1 b_1}\right)Z(s)\mathrm{d}s + \frac{\varphi_1(t)}{t}] - \frac{1}{t}\int_0^t bZ(s)\mathrm{d}s$$

$$+ \frac{1}{t}\sigma_2 B_2(t) + \frac{1}{t}M_2(t)$$

$$= R_0^* + \frac{a}{D}[a_1\mu_3 - (\mu_1 + D_1)]\frac{1}{t}\int_0^t P(s)\mathrm{d}s + \frac{aa_1\varphi_1}{Dt} + \frac{1}{t}\sigma_2 B_2(t)$$

$$+ [\frac{aa_1}{D}\left(\mu_4 - \frac{\mu_2 + D_2}{a_1 b_1}\right) - b]\frac{1}{t}\int_0^t Z(s)\mathrm{d}s + \frac{1}{t}M_2(t)$$

$$\leq R_0^* - \frac{a}{D}(\mu_1 + D_1 - a_1\mu_3)\frac{1}{t}\int_0^t P(s)\mathrm{d}s + \frac{aa_1\varphi_1}{Dt} + \frac{1}{t}\sigma_2 B_2(t) + \frac{1}{t}M_2(t).$$

This together with Lemma 2.2 yields to $\lim_{t\to\infty} P(t) = 0$ a.s. if $R_0^* = aa_1 N^0 - (\mu_1 + D_1 + \beta_2) < 0$. And then $\lim_{t\to\infty} Z(t) = 0$ a.s. by Lemma 2.1.

Now we prove (ii). If $R_0^* = aa_1 N^0 - (\mu_1 + D_1 + \beta_2) \geq 0$, we easily get

$$\langle P(t)\rangle^* \leq \frac{R_0^*}{\frac{a}{D}(\mu_1 + D_1 - a_1\mu_3)} = \frac{DR_0^*}{a(\mu_1 + D_1 - a_1\mu_3)}. \tag{4.6}$$

Substituting (4.6) into (4.4) leads to

$$\frac{1}{t}\ln\frac{Z(t)}{Z(0)} = -(\mu_2 + D_2 + \beta_3) + \frac{1}{t}\int_0^t bb_1 P(s)\mathrm{d}s + \frac{1}{t}\sigma_3 B_3(t) + \frac{1}{t}M_3(t)$$

$$\leq -(\mu_2 + D_2 + \beta_3) + bb_1\langle P(t)\rangle^* + \frac{1}{t}\sigma_3 B_3(t) + \frac{1}{t}M_3(t)$$

$$\leq -(\mu_2 + D_2 + \beta_3) + bb_1\frac{DR_0^*}{a(\mu_1 + D_1 - a_1\mu_3)} + \frac{1}{t}\sigma_3 B_3(t) + \frac{1}{t}M_3(t)$$

$$= \frac{R_1^*}{D(\mu_1 + D_1 - a_1\mu_3)} + \frac{1}{t}\sigma_3 B_3(t) + \frac{1}{t}M_3(t).$$

If $R_1^* < 0$, by Lemma 2.2 we have $\lim_{t\to\infty} Z(t) = 0$ a.s. which implies $\frac{1}{t}\int_0^t Z(s)ds <$

ε. Substituting this equation into (4.5), we have

$$
\begin{aligned}
\frac{1}{t}\ln\frac{P(t)}{P(0)} =& aa_1 N^0 - (\mu_1 + D_1 + \beta_2) + \frac{a}{D}[a_1\mu_3 - (\mu_1 + D_1)]\frac{1}{t}\int_0^t P(s)\mathrm{d}s \\
& + \left[\frac{aa_1}{D}\left(\mu_4 - \frac{\mu_2 + D_2}{a_1 b_1}\right) - b\right]\frac{1}{t}\int_0^t Z(s)\mathrm{d}s + \frac{aa_1\varphi_1(t)}{Dt} \\
& + \frac{1}{t}\sigma_2 B_2(t) + \frac{1}{t}M_2(t) \\
\geq& R_0^* - \frac{a}{D}(\mu_1 + D_1 - a_1\mu_3)\frac{1}{t}\int_0^t P(s)\mathrm{d}s + \left[\frac{aa_1}{D}\left(\mu_4 - \frac{\mu_2 + D_2}{a_1 b_1}\right) - b\right]\varepsilon \\
& + \frac{aa_1\varphi_1(t)}{Dt} + \frac{1}{t}\sigma_2 B_2(t) + \frac{1}{t}M_2(t).
\end{aligned}
$$

In virtue of (4.7), Lemma 2.2 and the arbitrariness of ε, we obtain

$$
\langle P(t)\rangle_* \geq \frac{R_0^*}{\frac{a}{D}(\mu_1 + D_1 - a_1\mu_3)} = \frac{DR_0^*}{a(\mu_1 + D_1 - a_1\mu_3)}. \tag{4.7}
$$

Combining (4.6) and (4.7), we have

$$
\langle P(t)\rangle = \lim_{t\to\infty}\frac{1}{t}\int_0^t P(s)\mathrm{d}s = \frac{DR_0^*}{a(\mu_1 + D_1 - a_1\mu_3)}.
$$

This completes the proof of theorem 4.1. $\qquad\square$

Remark 4.1. Theorem 4.1 clarifies the natural phenomenon that the maximum growth rate of phytoplankton must be no less than the combined effects of its own mortality, the washout rate of phytoplankton and stochastic environment noise, otherwise the phytoplankton must become extinct.

Next, we will study the persistence of plankton for the stochastic system (1.3).

Theorem 4.2. If $R_2^* \geq 0$, $\min\{Q_1, Q_2\} > 0$, then $\min\{\langle P(t)\rangle_*, \langle Z(t)\rangle_*\} > 0$, showing that both phytoplankton and zooplankton are persistent.

Proof.   In virtue of (4.5), we obtain

$$
\begin{aligned}
\left(\frac{\mu_1 + D_1}{a_1} - \mu_3\right)\frac{1}{t}\int_0^t P(s)\mathrm{d}s =& DN^0 - \frac{1}{t}\int_0^t\left(\frac{\mu_2 + D_2}{a_1 b_1} - \mu_4\right)Z(s)\mathrm{d}s \\
& - \frac{1}{t}\int_0^t DN(s)\mathrm{d}s + \frac{\varphi_1(t)}{t},
\end{aligned} \tag{4.8}
$$

where

$$
\begin{aligned}
\varphi_1(t) =& U(0) - U(N, P, Z) + \int_0^t \sigma_1 N(s)\mathrm{d}B_1(s) + \frac{\sigma_2}{a_1}\int_0^t P(s)\mathrm{d}B_2(s) \\
& + \frac{\sigma_3}{a_1 b_1}\int_0^t Z(s)\mathrm{d}B_3(s) + \int_0^t\int_{\mathbb{Y}}\gamma_1(u)N(s)\widetilde{\mathrm{N}}(\mathrm{d}s, \mathrm{d}u) \\
& + \frac{1}{a_1}\int_0^t\int_{\mathbb{Y}}\gamma_2(u)P(s)\widetilde{\mathrm{N}}(\mathrm{d}s, \mathrm{d}u) + \frac{1}{a_1 b_1}\int_0^t\int_{\mathbb{Y}}\gamma_3(u)Z(s)\widetilde{\mathrm{N}}(\mathrm{d}s, \mathrm{d}u).
\end{aligned}
$$

Based on $\lim_{t\to\infty} \frac{\varphi_1(t)}{t} = 0$, then substituting (4.8) into (4.4), one has

$$\frac{1}{t}\ln\frac{Z(t)}{Z(0)} = -(\mu_2 + D_2 + \beta_3) + \frac{bb_1}{\frac{\mu_1+D_1}{a_1} - \mu_3}[DN^0 + \frac{1}{t}\int_0^t \left(\mu_4 - \frac{\mu_2 + D_2}{a_1 b_1}\right) Z(s)\mathrm{d}s$$

$$- \frac{1}{t}\int_0^t DN(s)\mathrm{d}s + \frac{\varphi_1(t)}{t}] + \frac{1}{t}\sigma_3 B_3(t) + \frac{1}{t}M_3(t)$$

$$= R_2^* - \frac{b(\mu_2 + D_2 - a_1 b_1 \mu_4)}{\mu_1 + D_1 - a_1\mu_3}\frac{1}{t}\int_0^t Z(s)\mathrm{d}s - \frac{a_1 Dbb_1}{\mu_1 + D_1 - a_1\mu_3}\frac{1}{t}\int_0^t N(s)\mathrm{d}s$$

$$+ \frac{a_1 bb_1}{\mu_1 + D_1 - a_1\mu_3}\frac{\varphi_1(t)}{t} + \frac{1}{t}\sigma_3 B_3(t) + \frac{1}{t}M_3(t)$$

$$\leq R_2^* - \frac{b(\mu_2 + D_2 - a_1 b_1 \mu_4)}{\mu_1 + D_1 - a_1\mu_3}\frac{1}{t}\int_0^t Z(s)\mathrm{d}s + \frac{a_1 bb_1}{\mu_1 + D_1 - a_1\mu_3}\frac{\varphi_1(t)}{t}$$

$$+ \frac{1}{t}\sigma_3 B_3(t) + \frac{1}{t}M_3(t).$$

Clearly, if $R_2^* = \frac{a_1 b_1 bDN^0}{\mu_1 + D_1 - a_1\mu_3} - (\mu_2 + D_2 + \beta_3) \geq 0$, then

$$\langle Z(t)\rangle^* \leq \frac{R_2^*(\mu_1 + D_1 - a_1\mu_3)}{b(\mu_2 + D_2 - a_1 b_1 \mu_4)} = Q_0. \tag{4.9}$$

Substituting (4.9) into (4.6), we obtain

$$\frac{1}{t}\ln\frac{P(t)}{P(0)} \geq aa_1 N^0 - (\mu_1 + D_1 + \beta_2) + \frac{a}{D}[a_1\mu_3 - (\mu_1 + D_1)]\frac{1}{t}\int_0^t P(s)\mathrm{d}s$$

$$+ \left[\frac{aa_1}{D}(\mu_4 - \frac{\mu_2 + D_2}{a_1 b_1}) - b\right]\langle Z(t)\rangle^* + \frac{aa_1\varphi_1(t)}{Dt} + \frac{1}{t}\sigma_2 B_2(t) + \frac{1}{t}M_2(t)$$

$$\geq R_0^* + \frac{a}{D}[a_1\mu_3 - (\mu_1 + D_1)]\frac{1}{t}\int_0^t P(s)\mathrm{d}s + \left[\frac{aa_1}{D}(\mu_4 - \frac{\mu_2 + D_2}{a_1 b_1}) - b\right]Q_0$$

$$+ \frac{aa_1\varphi_1(t)}{Dt} + \frac{1}{t}\sigma_2 B_2(t) + \frac{1}{t}M_2(t)$$

$$= Q_1 - \frac{a}{D}(\mu_1 + D_1 - a_1\mu_3)\frac{1}{t}\int_0^t P(s)\mathrm{d}s + \frac{aa_1\varphi_1(t)}{Dt} + \frac{1}{t}\sigma_2 B_2(t) + \frac{1}{t}M_2(t),$$

where $Q_1 = R_0^* + \left[\frac{aa_1}{D}(\mu_4 - \frac{\mu_2 + D_2}{a_1 b_1}) - b\right]Q_0$. If $Q_1 > 0$, in view of Lemma 2.2, one has

$$\langle P(t)\rangle_* \geq \frac{Q_1}{\frac{a}{D}(\mu_1 + D_1 - a_1\mu_3)} = \frac{DQ_1}{a(\mu_1 + D_1 - a_1\mu_3)} > 0. \tag{4.10}$$

On the other hand, substituting (4.10) into (4.4), we can obtain

$$\frac{1}{t}\ln\frac{Z(t)}{Z(0)} = -(\mu_2 + D_2 + \beta_3) + \frac{1}{t}\int_0^t bb_1 P(s)ds + \frac{1}{t}\sigma_3 B_3(t) + \frac{1}{t}M_3(t)$$

$$\geq -(\mu_2 + D_2 + \beta_3) + bb_1\langle P(t)\rangle_* + \frac{1}{t}\sigma_3 B_3(t) + \frac{1}{t}M_3(t)$$

$$\geq -(\mu_2 + D_2 + \beta_3) + \frac{bb_1 DQ_1}{a(\mu_1 + D_1 - a_1\mu_3)} + \frac{1}{t}\sigma_3 B_3(t) + \frac{1}{t}M_3(t).$$

Obviously, if $Q_2 > 0$, where Q_2 is defined in (4.1), we have $\langle Z(t)\rangle_* > 0$, which implies that the zooplankton is persistent. This completes the proof. □

Remark 4.2. Based on Theorem 4.2, we easily obtain that if the zooplankton gets more energy by feeding on phytoplankton than it consumes for its own growth and environment noise, then the zooplankton is persistent. Whereas phytoplankton need to support not only their own growth and environmental noise, but also the impact of zooplankton on them.

5. Numerical Simulations

In this section, we will numerically illustrate the effects of environmental noise on the dynamics of plankton.

Example 5.1. We choose the initial values $(N(0), P(0), Z(0)) = (1, 1, 1)$ and take the other parameters in model (1.3) as $D = 0.2$, $N^0 = 2$, $a = 0.6$, $b = 0.5$, $D_1 = 0.03$, $D_2 = 0.1$, $a_1 = 0.5$, $b_1 = 0.6$, $\mu_1 = 0.01$, $\mu_2 = 0.01$, $\mu_3 = 0.008$, $\mu_4 = 0.005$, $\sigma_1 = 0.1$, $\sigma_2 = 0.6$, $\sigma_3 = 0.02$, $\gamma_1 = 0.2$, $\gamma_2 = -0.1$ and $\gamma_3 = 0.2$.

By simple calculations, we have $R_0^* = -0.2944 < 0$, which shows that both phytoplankton and zooplankton tend to be extinct, as is shown in Fig. 1.
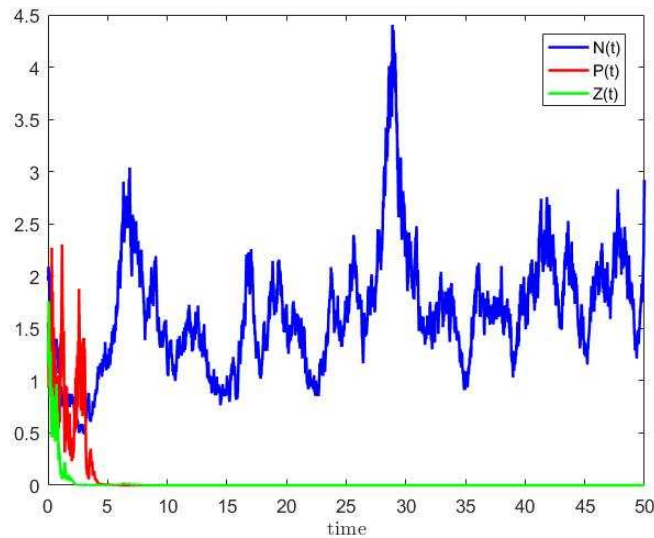

Figure 1. Sample paths of nutrient, phytoplankton and zooplankton, respectively. Here both phytoplankton and zooplankton tend to be extinct.

Example 5.2. We choose the initial values $(N(0), P(0), Z(0)) = (2, 1.5, 1.2)$ and model parameters $D = 0.2$, $N^0 = 6$, $a = 0.6$, $b = 0.5$, $D_1 = 0.03$, $D_2 = 0.1$, $a_1 = 0.5$, $b_1 = 0.6$, $\mu_1 = 0.1$, $\mu_2 = 0.1$, $\mu_3 = 0.08$ and $\mu_4 = 0.05$.

With this setting, we illustrate the effects of Lévy jumps by varying the intensity of noise. Firstly, let $\sigma_1 = 0.1$, $\sigma_2 = 0.002$, $\sigma_3 = 0.02$, $\gamma_1 = 0.2$, $\gamma_2 = 0.04$ and $\gamma_3 = -0.1$. Then, $R_0^* = 1.6692 > 0$ and $R_1^* = -0.013248 < 0$. By Theorem 4.1, the phytoplankton is persistent and the zooplankton tends to be extinct, as is shown in Fig. 2.

We next change the intensity of noise to $\sigma_1 = 0.1$, $\sigma_2 = 0.02$, $\sigma_3 = 0.6$, $\gamma_1 = 0.2$, $\gamma_2 = 0.2586$, $\gamma_3 = -0.1$ and keep the other parameters unchanged. By computing, we have $R_2^* = 1.6146 > 0$, $Q_1 = 1.4267 > 0$ and $Q_2 = 4.911 > 0$, which implies the persistence of both phytoplankton and zooplankton, as is shown in Fig. 3.
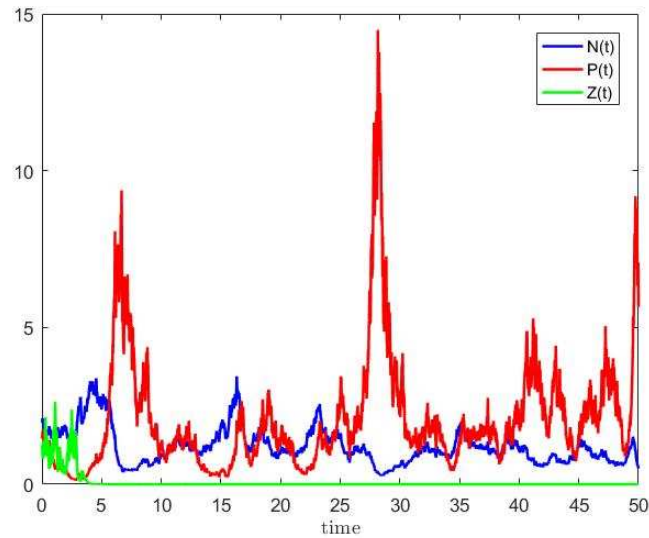

Figure 2. Sample paths of nutrient, phytoplankton and zooplankton, respectively. Here phytoplankton is persistent and zooplankton is extinct.
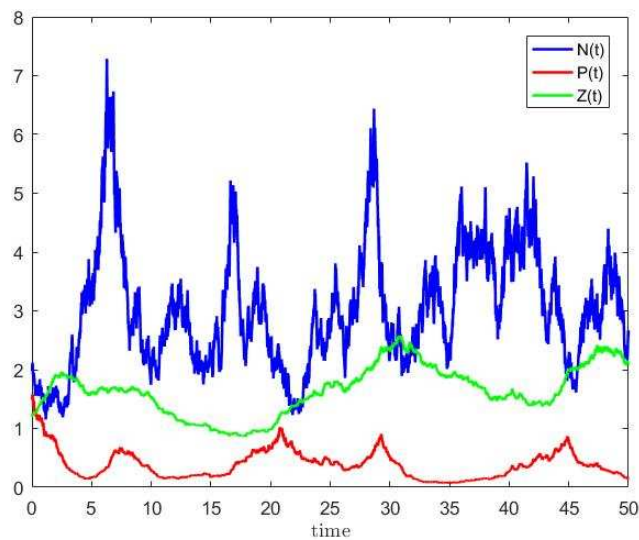

Figure 3. Sample paths of nutrient, phytoplankton and zooplankton, respectively. Here both phytoplankton and zooplankton tend to be persistent.

By comparing Fig. 1, Fig. 2 and Fig. 3, one can find that the survival of phytoplankton depends on the effect of environmental noise on itself, and the survival of zooplankton depends on the effects of environmental noise on itself and phytoplankton, while nutrient is always persistent regardless of the intensity of the noise. These results reveal that the Lévy noise can change the properties of the plankton system significantly.

6. Conclusion and discussion

In natural ecology, there are many complex relationships and interactions between organisms, which constitute a biological population system together [29]. And environmental noise is ubiquitous in nature and has a significant impact on population dynamics [30,31]. To better understand the interactions between nutrients and food webs, we propose a stochastic nutrient-plankton food chain model with Lévy jumps and study its dynamics. Firstly, by constructing a suitable Lyapunov function, we prove that the model has a unique global positive solution for any given positive initial value. Then, we establish sufficient conditions for the persistence and extinction of plankton. Finally, some examples together with numerical simulations were provided to illustrate the analytical results.

For the corresponding stochastic model (1.2), the authors in [32] have proved that there are two thresholds, $R_0^s = aN^0 - (\mu_1 + D_1 + \frac{1}{2}\sigma_2^2)$ and $R_1^s = cD(aN^0 - \mu_1 - D_1 - \frac{1}{2}\sigma_2^2) - a(\mu_2 + D_2 + \frac{1}{2}\sigma_3^2)(\mu_1 + D_1 - \mu_3)$, which determine the persistence and extinction of plankton. In this sense, stochastic model (1.3) extends the existing stochastic one and our results improve the corresponding researches. Theoretical results show that

i if $R_0^* = a_1 aN^0 - (\mu_1 + D_1 + \beta_2) < 0$, both phytoplankton and zooplankton tend to be extinct, as is shown in Fig. 1;

ii if $R_0^* \geq 0$ and $R_1^* = Dbb_1(aa_1N^0 - \mu_1 - D_1 - \beta_2) - a(\mu_2 + D_2 + \beta_3)(\mu_1 + D_1 - a_1\mu_3) < 0$, the phytoplankton is persistent and the zooplankton tends to be extinct, as is shown in Fig. 2;

iii if $R_2^* \geq 0$, $Q_1 > 0$ and $Q_2 > 0$, where R_2^*, Q_1, Q_2 are defined in (4.1), then both the phytoplankton and the zooplankton are persistent, as is shown in Fig. 3.

We can also compare the stochastic system (1.2) R_0^s and R_1^s, and introduce the effects of the Lévy noise into this system and the model(1.3) is an extension of the corresponding model (1.2). These results reveal that the Lévy noise can change the properties of the plankton system significantly and it can force the phytoplankton and the zooplankton to become extinct. Hence, the Lévy noise in marine ecosystem can seriously affect the dynamics of plankton and cannot be ignored.

The goal of the paper is to formulate a stochastic nutrient-phytoplankton-zooplankton model for understanding the effects of environmental fluctuations on the dynamics of plankton. The obtained results enrich the dynamics research of the nutrient-plankton model, which can help us better understand the interaction between them in stochastic sense. There are many interesting issues to do in the future. On one hand, one may obtain the ergodic stationary distribution of the stochastic nutrient-plankton food chain model with Lévy jumps. On the other hand, based on the model again, whether zooplankton and phytoplankton will cause periodic changes due to seasons. We leave these for future investigation.

Acknowledgments

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