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The Upper Semicontinuity of Random Attractor for Stochastic Suspension Bridge Equation*

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Abstract Based on the existence of pullback attractors for stochastic suspension bridge in [7], in the paper, we further investigate the upper semicontinuity of pullback attractors for the problem.

Keywords Upper semicontinuity, Suspension bridge equations, Random attractors, Linear memory, Additive noise.

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1. Introduction

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, where

$$\Omega = \{\omega = (\omega_1, \omega_2, \cdots, \omega_m) \in C(\mathbb{R}, \mathbb{R}^m) : \omega(0) = 0\},$$

is endowed with compact open topology, $\mathcal{F}$ is the $\mathbb{P}$-completion of Borel σ-algebra on Ω, and $\mathbb{P}$ is the corresponding Wiener measure. Define the time shift via

$$\theta_t\omega(\cdot) = \omega(\cdot + t) - \omega(t), \quad t \in \mathbb{R}, \ \omega \in \Omega.$$

Thus, $(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ is an ergodic metric dynamical system.

In this paper, we are devoted to considering the upper semicontinuity of random attractors for the following suspension bridge equations with linear memory and additive white noise:

$$\begin{cases} u_{tt} + \Delta^2 u + \Delta^2 u_t + ku^+ + (p - \beta\|\nabla u\|^2_{L^2(U)})\Delta u + \int_0^\infty \mu(s)\Delta^2(u(t) \\ \qquad - u(t-s))ds = g(x) + \alpha \sum_{j=1}^m h_j \dot{W}_j, \ x \in U, t > \tau, \\ u(x,t) = \Delta u(x,t) = 0, \qquad x \in \partial U, \ t \le \tau, \ \tau \in \mathbb{R}, \\ u(x,\tau) = u_0(x), u_t(x,\tau) = u_1(x), \qquad x \in U, \end{cases} \tag{1.1}$$

where U is a bounded open set of $\mathbb{R}^2$ with a smooth boundary ∂U, $u = u(x,t)$ is a real-valued function on $U \times [\tau, +\infty)$ and accounts for the downward deflection of

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the bridge in the vertical plane, u^+ namely stands for its positive part,

$$u^+ = \begin{cases} u, & \text{if } u \geq 0, \\ 0, & \text{if } u \leq 0. \end{cases}$$

$k > 0$ denotes the spring constant, and α is a positive constant. The real constant p represents the axial force acting at the end of the road bed of the bridge in the reference configuration. Namely, p is negative when the bridge is stretched, positive when compressed, $h_j(x) \in H_0^2(U) \cap H^4(U)$, $(j = 1, 2, 3, \cdots, m)$, $\{W_j\}_{j=1}^m$ are independent two-sided real-valued Wiener processes on $(\Omega, \mathfrak{F}, \mathbb{P})$. We identify $\omega(t)$ with $(W_1(t), W_2(t), \cdots, W_m(t))$, i. e.,

$$\omega(t) = (W_1(t), W_2(t), \cdots, W_m(t)), t \in \mathbb{R}.$$

The memory kernel function $\mu(s)$ and $g(x)$ satisfy the following conditions:

(H_1) : $\mu(s) \in C^1(\mathbb{R}^+) \cap L^2(\mathbb{R}^+)$, $\mu(s) \geq 0$, $\mu'(s) + \delta\mu(s) \leq 0$, $\forall s \in \mathbb{R}^+$ and some $\delta > 0$.

(H_2) : $g \in H_0^1 \cap H^2(U)$.

Following Dafermos [1], we introduce a Hilbert "history" space

$$\mathfrak{R}_{\mu,2} = L^2_\mu(\mathbb{R}^+, H^2(U) \cap H_0^1(U))$$

with the inner product

$$(\eta_1, \eta_2)_{\mu,2} = \int_0^\infty \mu(s) \left(\Delta\eta_1(s), \Delta\eta_2(s)\right) ds, \quad \forall \eta_1, \eta_2 \in \mathfrak{R}_{\mu,2},$$

and new variables

$$\eta(t, x, s) = u(t, x) - u(t - s, x).$$

To facilitate easy calculation, we take $\beta = 1$. Then, we set $E = (H^2(U) \cap H_0^1(U)) \times L^2(U) \times \mathfrak{R}_{\mu,2}$, $Z = (u, u_t, \eta)^T$. Then, the system (1.1) is equivalent to the following initial value problem in the Hilbert space E :

$$\begin{cases} Z_t = L(Z) + N(Z, t, W(t)), & x \in U,\ t \geq \tau,\ s \in \mathbb{R}^+, \\ Z(\tau) = Z_\tau = (u_0(x), u_1(x), \eta_0(x, s)), & (x, s) \in U \times \mathbb{R}^+, \end{cases} \tag{1.2}$$

where

$$\begin{cases} u(t, \tau, x) = \eta(t, \tau, x, s) = \eta(t, \tau, x, 0) = 0,\ x \in \partial U,\ t > \tau,\ s \in \mathbb{R}^+, \\ \Delta u(t, \tau, x) = \Delta\eta(t, \tau, x, s) = \Delta\eta(t, \tau, x, 0) = 0,\ x \in \partial U,\ t \leq \tau,\ s \in \mathbb{R}^+, \\ u(\tau, x) = u(\tau, \tau, x) = u_0(\tau, x),\ u_t(\tau, x) = u_t(\tau, \tau, x) = u_1(x), \qquad x \in U, \\ \eta(\tau, x, s) = \eta_0(x, s) = u(\tau, x) - u(\tau - s, x), \qquad (x, s) \in U \times \mathbb{R}^+, \end{cases} \tag{1.3}$$

$$L(Z) = \begin{pmatrix} u_t \\ -\Delta^2 u - \Delta^2 u_t - \int_0^\infty \mu(s)\Delta^2\eta(s)ds \\ u_t - \eta_s \end{pmatrix}, \tag{1.4}$$

$$N(Z,t,W(t)) = \begin{pmatrix} 0 \\ -ku^{+} - (p - \|\nabla u\|^2_{L^2(U)})\Delta u + \alpha \sum_{j=1}^{m} h_j \dot{W}_j \\ 0 \end{pmatrix}, \tag{1.5}$$

$$D(L) = \left\{ Z \in E \mid \begin{array}{c} u + \int_0^\infty \mu(s)\Delta^2 \eta(s) ds \in H^3(U) \cap H_0^2(U), \\ u_t \in H_0^2(U), \eta(s) \in H_\mu^1(\mathbb{R}^+, H^2(U) \cap H_0^1(U)), \eta(\tau) = 0 \end{array} \right\}. \tag{1.6}$$

Here, $H_\mu^1(\mathbb{R}^+, H^2(U) \cap H_0^1(U)) = \{\eta : \eta(s), \partial_s \eta(s) \in L_\mu^2(\mathbb{R}^+, H^2(U) \cap H_0^1(U))\}$.

According to the [7], the problem (1.2) is equivalent to the following determined system with random parameters in E :

$$\dot{\varphi} + H(\varphi) = F(\varphi, \theta_t \omega, t), \quad \varphi_\tau(\omega) = (u_0, u_1 + \varepsilon u_0 - \alpha z(\theta_\tau \omega), \eta_0)^T, \ t \geq \tau, \ \tau \in \mathbb{R}, \tag{1.7}$$

where

$$F(\varphi, \theta_t \omega, t) = \begin{pmatrix} z(\theta_t \omega) \\ -ku^{+} - (p - \|\nabla u\|^2)\Delta u + g(x) + \alpha z(\theta_t \omega) \\ z(\theta_t \omega) \end{pmatrix}. \tag{1.8}$$

In [2], the authors obtained the existence of a compact random attractor for the random dynamical system generated by the coupled suspension bridge equations with white noises. While in [5], random attractors for the stochastic coupled suspension bridge equations of Kirchhoff type were studied. The upper semicontinuity of random attractors for the stochastic non-autonomous suspension bridge equation with memory was established in [6] without Kirchhoff type. Just for problem (1.1), the author investigated the random attractors for stochastic suspension bridge equation with additive noise (see [7] for details). To the best of our knowledge, it is not considered by any predecessors for the upper semicontinuity of pullback attractors for the problem (1.1). Based on the results in [7], the theory and applications of B. Wang in [3, 4], we decide to study the upper semicontinuity of pullback attractors for problem (1.1).

The rest of this paper is organized as follows: In the next section, we present some notations, definitions and a criteria concerning the upper semicontinuity of random attractors with respect to a parameter. In Section 3, we show the upper semi-continuity of random attractors.

Throughout the paper, we apply $||\cdot||$ and $(\cdot,\cdot)$ to denote the norm and the inner product of $L^2(\mathbb{R}^n)$ respectively. The norms of $L^p(\mathbb{R}^n)$ and a Banach space X are generally written as $||\cdot||_p$ and $||\cdot||_X$ respectively. The letters c and c_i $(i = 1, 2, \dots)$ are generic positive constants, which may change their values from line to line or even in the same line, and do not depend on α.

2. Preliminaries

In this section, we first present some notations. Then, we recall some definitions and known results regarding non-autonomous random dynamical systems from [4], which are useful to our problem.

From now on, assume that conditions $(H_1)-(H_2)$ hold, the space E, $(\Omega,\mathcal{F},\mathbb{P},(\theta_t)_{t\in\mathbb{R}})$ are defined as in Section 1. With the usual notation, we denote $A=\triangle^2$, $A^{\frac{1}{2}}=-\triangle$ and $D(A)=\{u\in H^2(U)\cap H_0^1(U)\ |Au\in L^2(U)\}$. We can define $\mathcal{H}^r=D(A^{\frac{r}{4}})$. The space defined above is a Hilbert space with the following inner product and norm

$$(u,v)_r=(A^{\frac{r}{4}}u,A^{\frac{r}{4}}v),\ \|\cdot\|_r=\|A^{\frac{r}{4}}u\|,\ \ \forall\ u,v\in\mathcal{H}^r.$$

In particular, $D(A^0)=L^2(U)$, $D(A^{\frac{1}{2}})=H^2(U)\cap H_0^1(U)$. The inner product and norm in $L^2(U)$ is denoted by $(\cdot,\cdot)$, $\|\cdot\|$, and in $H^2(U)\cap H_0^1(U)$ is denoted by $((\cdot,\cdot))$, $\|\cdot\|_2$. By (H_1), the space $\mathfrak{R}_{\mu,r}=L^2_\mu(\mathbb{R}^+,\mathcal{H}^r)$ is a Hilbert space with the inner product and norm respectively

$$\begin{aligned}(\eta,\eta_1)_{\mu,r}&=\int_0^\infty\mu(s)(A^{\frac{r}{4}}\eta(s),A^{\frac{r}{4}}\eta_1(s))ds,\\ \|\eta\|^2_{\mu,r}&=\int_0^\infty\mu(s)(A^{\frac{r}{4}}\eta(s),A^{\frac{r}{4}}\eta(s))ds,\end{aligned}\qquad\forall\ \eta,\ \eta_1\in\mathcal{H}^r,$$

the linear operator $-\partial_s$ has domain

$$D(-\partial_s)=\{\eta\in H^1_\mu(\mathbb{R}^+,\mathcal{H}^r):\eta(0)=0\},$$

where $H^1_\mu(\mathbb{R}^+,\mathcal{H}^r)=\{\eta:\eta(s),\partial_s\eta(s)\in L^2_\mu(\mathbb{R}^+,\mathcal{H}^r)\}$, which generates a right-translation semigroup. The symbol C and $C_i(i=1,2,\cdots)$ are used for a general positive number which may change from line to line.

Definition 2.1. Let $\theta:\mathbb{R}\times\Omega\to\Omega$ be a $(\mathcal{B}(\mathbb{R})\times\mathcal{F},\mathcal{F})$-measurable mapping. We say $(\Omega,\mathcal{F},\mathcal{P},\theta)$ is a parametric dynamical system if $\theta(0,\cdot)$ is the identity on Ω, $\theta(s+t,\cdot)=\theta(t,\cdot)\circ\theta(s,\cdot)$ for all $t,s\in\mathbb{R}$, and $P\theta(t,\cdot)=P$ for all $t\in\mathbb{R}$.

Definition 2.2. Let $K:\mathbb{R}\times\Omega\to 2^X$ be a set-valued mapping with closed nonempty images. We say K is measurable with respect to $\mathcal{F}$ in Ω if the mapping $\omega\in\Omega\to d(x,K(\tau,\omega))$ is $(\mathcal{F},\mathcal{B}(\mathbb{R}))$-measurable for every fixed $x\in X$ and $\tau\in\mathbb{R}$.

Definition 2.3. A mapping $\Phi:\mathbb{R}^+\times\mathbb{R}\times\Omega\times X\to X$ is called a continuous cocycle on X over $\mathbb{R}$ and $(\Omega,\mathcal{F},\mathcal{P},\{\theta_t\}_{t\in\mathbb{R}})$ if for all $\tau\in\mathbb{R}$, $\omega\in\Omega$ and $t,s\in\mathbb{R}^+$, the following conditions (1)-(4) are satisfied:
(1) $\Phi(\cdot,\tau,\cdot,\cdot):\mathbb{R}^+\times\Omega\times X\to X$ is $(\mathcal{B}(\mathbb{R}^+)\times\mathcal{F}\times\mathcal{B}(X),\mathcal{B}(X))$-measurable;
(2) $\Phi(0,\tau,\omega,\cdot)$ is the identity on X;
(3) $\Phi(t+s,\tau,\omega,\cdot)=\Phi(t,\tau+s,\theta_s\omega,\cdot)\circ\Phi(s,\tau,\omega,\cdot)$;
(4) $\Phi(t,\tau,\omega,\cdot):X\to X$ is continuous.

Hereafter, we assume Φ is a continuous cocycle on X over $\mathbb{R}$ and $(\Omega,\mathcal{F},\mathcal{P},\{\theta_t\}_{t\in\mathbb{R}})$, and $\mathcal{D}$ is the collection of some families of nonempty bounded subsets of X parameterized by $\tau\in\mathbb{R}$ and $\omega\in\Omega$:

$$\mathcal{D}=\{D=\{D(\tau,\omega)\subseteq X:D(\tau,\omega)\neq\emptyset,\tau\in\mathbb{R},\omega\in\Omega\}\}.$$

Definition 2.4. Let $\mathcal{D}$ be a collection of some families of nonempty subsets of X and $K=\{K(\tau,\omega):\tau\in\mathbb{R},\omega\in\Omega\}\in\mathcal{D}$. Then, K is called a $\mathcal{D}$-pullback absorbing set for Φ if for all $\tau\in\mathbb{R}$ and $\omega\in\Omega$ and for every $B\in\mathcal{D}$, there exists $T=T(B,\tau,\omega)>0$ such that

$$\Phi(t,\tau-t,\theta_{-t}\omega,B(\tau-t,\theta_{-t}\omega))\subseteq K(\tau,\omega)\ \text{ for all } t\geq T.$$

In addition, if $K(\tau,\omega)$ is closed in X and is measurable in ω with respect to $\mathcal{F}$, then K is called a closed measurable $\mathcal{D}$-pullback absorbing set for Φ.

Definition 2.5. Let $\mathcal{D}$ be a collection of some families of nonempty subsets of X and $\mathcal{A}=\{\mathcal{A}(\tau,\omega):\tau\in\mathbb{R},\omega\in\Omega\}\in\mathcal{D}$. Then, $\mathcal{A}$ is called a $\mathcal{D}$-pullback attractor for Φ if the following conditions (1)-(3) are fulfilled: for all $t\in\mathbb{R}^+$, $\tau\in\mathbb{R}$ and $\omega\in\Omega$,

(1) $\mathcal{A}(\tau,\omega)$ is compact in X and is measurable in ω with respect to $\mathcal{F}$.

(2) $\mathcal{A}$ is invariant, that is,

$$\Phi(t,\tau,\omega,\mathcal{A}(\tau,\omega))=\mathcal{A}(\tau+t,\theta_t\omega).$$

(3) For every $B=\{B(\tau,\omega):\tau\in\mathbb{R},\omega\in\Omega\}\in\mathcal{D}$,

$$\lim_{t\to\infty} d(\Phi(t,\tau-t,\theta_{-t}\omega,B(\tau-t,\theta_{-t}\omega)),\mathcal{A}(\tau,\omega))=0. \tag{2.1}$$

Finally, we present a criteria concerning the upper semicontinuity of random attractors with respect to a parameter.

Theorem 2.1. *Let $(X,\|\cdot\|_X)$ be a separable Banach space, Φ_α be a continuous cocycle on X over $\mathbb{R}$ and $(\Omega,\mathcal{F},\mathcal{P},\{\theta_t\}_{t\in\mathbb{R}})$. Suppose that*
(i) Φ_α has a closed measurable random absorbing set $K_\alpha=\{K_\alpha(\tau,\omega):\tau\in\mathbb{R},\omega\in\Omega\}$ in $\mathcal{D}(X)$ and a unique random attractor $\mathcal{A}_\alpha=\{\mathcal{A}_\alpha(\tau,\omega):\tau\in\mathbb{R},\omega\in\Omega\}$ in $\mathcal{D}(X)$.
(ii) There exists a map $\varsigma:\mathbb{R}\to\mathbb{R}$ such that for each $\tau\in\mathbb{R},\omega\in\Omega,K_0(\tau)=\{u\in X:\|u\|_X\le\varsigma(\tau)\}$ and

$$\limsup_{\alpha\to 0}\|K_\alpha(\tau,\omega)\|_X=\limsup_{\alpha\to 0}\ \limsup_{x\in K_\alpha(\tau,\omega)}\|x\|_X\le\varsigma(\tau). \tag{2.2}$$

(iii) There exists $\alpha_0>0$, such that for every $\tau\in\mathbb{R}$ and $\omega\in\Omega$,

$$\bigcup_{\alpha\le\alpha_0}\mathcal{A}_\alpha(\tau,\omega)\ \textit{is precompact in } X.$$

(iv) For $t>0,\tau\in\mathbb{R},\omega\in\Omega,\varepsilon_n\to 0$ when $n\to\infty$, and $x_n,x_0\in X$ with $x_n\to x_0$ when $n\to\infty$, it holds

$$\lim_{n\to\infty}\Phi_{\alpha_n}(t,\tau,\omega)x_n=\Phi_0(t,\tau)x_0. \tag{2.3}$$

Then, for $\tau\in\mathbb{R},\omega\in\Omega$,

$$d_H(\mathcal{A}_\alpha(\tau,\omega),\mathcal{A}_0(\tau))=\sup_{u\in\mathcal{A}_\alpha(\tau,\omega)}\ \inf_{v\in\mathcal{A}_0(\tau)}\|u-v\|\to 0,\quad \textit{as } \alpha\to 0. \tag{2.4}$$

3. Upper semicontinuity of pullback attractors

In this section, we consider the upper semicontinuity of pullback attractors for the stochastic equation (1.1).

Next, we use Theorem 2.1 to prove the upper semicontinuity of random attractors $\mathcal{A}_\alpha(\tau,\omega)$ when $\alpha\to 0$.

When $\alpha=0$, the system (1.7)-(1.8) reduces to a deterministic one in E:

$$\dot{\tilde{\varphi}}+H(\tilde{\varphi})=F(\tilde{\varphi},\theta_t\omega,t),\quad \tilde{\varphi}_\tau(\omega)=(\tilde{u}_0,\tilde{u}_1+\varepsilon\tilde{u}_0,\tilde{\eta}_0)^T,\ t\ge\tau,\ \tau\in\mathbb{R}, \tag{3.1}$$

where

$$F(\tilde{\varphi}, \theta_t \omega, t) = \begin{pmatrix} 0 \\ -k\tilde{u}^+ - (p - \|\nabla \tilde{u}\|^2)\Delta \tilde{u} + g(x) \\ 0 \end{pmatrix}. \tag{3.2}$$

Accordingly, by virtue of Theorem 6.2 in [7], the deterministic non-autonomous system Φ_0 generated by (3.1)-(3.2) is readily verified to admit a unique $\mathcal{D}_0(E)$-pullback attractor $\mathcal{A}_0(\tau)$.

Theorem 3.1. *Assume that (H1)-(H2) hold. Then for $\tau \in \mathbb{R}, \omega \in \Omega$,*

$$d_H(\mathcal{A}_\alpha(\tau,\omega), \mathcal{A}_0(\tau)) = \sup_{u \in \mathcal{A}_\alpha(\tau,\omega)} \inf_{v \in \mathcal{A}_0(\tau)} \|u - v\| \to 0, \quad \textit{as } \alpha \to 0.$$

Proof. (i) From Lemma 4.1 and Theorem 6.2 in [7], we know that Φ_ϵ has a closed measurable random absorbing set $E_\alpha = \{E_\alpha(\tau,\omega) : \tau \in \mathbb{R}, \omega \in \Omega\} \in \mathcal{D}\big(E\big)$, where $E_\alpha(\tau,\omega) = \big\{\varphi^{(\varepsilon)} \in E(\mathbb{R}^n) : \|\varphi^{(\alpha)}\|_E^2 \le r_1(\alpha,\tau,\omega)\big\}$, and a unique random attractor $\mathcal{A}_\alpha = \{\mathcal{A}_\alpha(\tau,\omega) : \tau \in \mathbb{R}, \omega \in \Omega\} \in \mathcal{D}(E)$, for each $\tau \in \mathbb{R}, \omega \in \Omega$, $\mathcal{A}_\alpha(\tau,\omega) \subseteq E_\alpha(\tau,\omega)$.

(ii) Given $\alpha \le 1$, by (4.1) in [7], we have

$$r_1(\alpha,\tau,\omega) \le r_1(1,\tau,\omega) < \infty,$$

and

$$\limsup_{\alpha \to 0} r_1(\alpha,\tau,\omega) \le r_1(1,\tau,\omega).$$

Therefore, for every $\tau \in \mathbb{R}, \omega \in \Omega$,

$$\limsup_{\alpha \to 0} \|E_\alpha(\tau,\omega)\| = \limsup_{\alpha \to 0} \sup_{x \in E_\alpha(\tau,\omega)} \|x\|_E \le r_1^{\frac{1}{2}}(1,\tau,\omega). \tag{3.3}$$

Let $E_1(\tau,\omega) = \{\varphi^{(\alpha)} \in E : \|\varphi^{(\alpha)}\|_E^2 \le r_1(1,\tau,\omega)\}$, then

$$\bigcup_{\alpha \le 1} \mathcal{A}_\alpha(\tau,\omega) \subseteq \bigcup_{\alpha \le 1} E_\alpha(\tau,\omega) \subseteq E_1(\tau,\omega). \tag{3.4}$$

(iii) Given $\alpha \le 1$, let us prove the precompactness of $\bigcup_{\alpha \le 1} \mathcal{A}_\alpha(\tau,\omega)$ for every $\tau \in \mathbb{R}, \omega \in \Omega$. By the invariance of $\mathcal{A}_\alpha(\tau,\omega)$, for every $\eta > 0, \alpha > 0, \tau \in \mathbb{R}, \omega \in \Omega$, such that the solution $\varphi^{(\alpha)}$ of (1.1) satisfies

$$\sup_{\varphi^{(\alpha)} \in \bigcup_{\alpha \le 1} \mathcal{A}_\alpha(\tau,\omega)} \big\|\varphi^{(\alpha)}\big(\tau, \tau - t, \theta_{-\tau}\omega, \varphi_0^{(\alpha)}\big)\big\|_E^2 \le \eta.$$

Therefore, we obtain that the set $\bigcup_{\alpha \le 1} \mathcal{A}_\alpha(\tau,\omega)$ is precompact in E.

(iv) Let $\alpha \le 1$, for every $\tau \in \mathbb{R}, \omega \in \Omega, t \ge 0$ and $r \ge \tau - t$, suppose $\varphi^{(\alpha)}$ and $\tilde{\varphi}$ are the solutions of (1.7)-(1.8) and (3.1)-(3.2) with corresponding initial data, respectively. Let $\hat{\varphi} = \varphi^{(\alpha)} - \tilde{\varphi} = (\tilde{u}, \tilde{v})$.

Similar to Lemma 4.1 in [7], by calculating, we can get the following:

$$\|\varphi^{(\alpha)}(\tau, \tau - t, \omega, \varphi_{\tau-t}^{(\alpha)}(\theta_{-\tau}\omega)) - \tilde{\varphi}(\tau, \tau - t, \tilde{\varphi}_{\tau-t}\|_E^2 \le r_2^2(\tau,\omega),$$

where $r_2^2(\tau, \omega)$ is a random variable.

For any $\tau \in \mathbb{R}, \omega \in \Omega, t \geq 0, \alpha_n \to 0$, it holds that

$$\lim_{\alpha_n \to 0} \varphi^{(\alpha_n)}(\tau, \tau - t, \omega, \varphi_{\tau-t}^{(\alpha_n)}(\theta_{-\tau}\omega) = \tilde{\varphi}(\tau, \tau - t, \tilde{\varphi}_{\tau-t}).$$

which along with (i), (ii), (iii) and Theorem 2.1 complete the proof. □

4. Conclusion

We overcome the difficulty by using a criteria concerning the upper semicontinuity of random attractors with respect to a parameter, and obtain the upper semicontinuity of random attractors for the problem (1.1).

Remark 4.1. I consider $\beta \neq 1$ that in equation (1.1), and I think it will be an interesting problem.

Remark 4.2. The motivation for considering the problem is to facilitate numerical calculation and simulation.

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