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Solitary Waves for the Generalized Nonautonomous Dual-power Nonlinear Schrödinger Equations with Variable Coefficients*

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Abstract In this paper, we study the solitary waves for the generalized nonautonomous dual-power nonlinear Schrödinger equations (DPNLS) with variable coefficients, which could be used to describe the saturation of the nonlinear refractive index and the solitons in photovoltaic-photorefractive materials such as LiNbO3, as well as many nonlinear optics problems. We generalize an explicit similarity transformation, which maps generalized nonautonomous DPNLS equations into ordinary autonomous DPNLS. Moreover, solitary waves of two concrete equations with space-quadratic potential and optical super-lattice potential are investigated.

Keywords Solitary waves, dual-power law, nonlinear Schrödinger equation, variable coefficients.

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1. Introduction

Phenomena in nonlinear optics and the Bose-Einstein condensates(BECs) are often described by nonlinear Schrödinger equations(NLS) [4–6, 9–11, 14, 16–18, 20, 23, 28]. For example, the wave phenomena observed in fluid dynamics, plasma and elastic media and optical fibers, etc. When we want to understand the physical mechanism of phenomena, exact solutions for the nonlinear Schrödinger equations have to be explored. Moreover, various types of the NLS with non-Kerr nonlinearities, which contain Kerr law, power law, parabolic law, dual-power law as well as the logarithmic law, and other varying potentials were studied by many researchers in [1–3, 7, 11–13, 15, 19, 22, 24–27, 30].

In this paper, we consider solitary waves of the 1D generalized nonautonomous dual-power nonlinear Schrödinger equations with variable coefficients(DPNLS)

$$\mathrm{i}Q_t + D(x,t)Q_{xx} + (lR_1(x,t)|Q|^n + kR_2(x,t)|Q|^{2n})Q + V(x,t)Q = 0, \tag{1.1}$$

where $Q(x,t)$ is the complex envelope of the propagating beam of the modes, x is the propagation distance, and t is the retarded time, l, n, k are arbitrary constants,

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$D(x,t)$ is the dispersion coefficient, $R_1(x,t), R_2(x,t)$ are the dual-power nonlinearity coefficients, respectively, and $V(x,t)$ is the external potential. This model contains many special types of the NLS with variable coefficients such as the cubic NLS equation, the cubic-quintic(CQ) NLS model, the generalized NLS model, etc [28]. Let $n = 2, k = 0$, l is still arbitrary constants, (1.1) collapses to the Kerr law nonlinear Schrödinger equations with variable coefficients

$$\mathrm{i}Q_t + D(x,t)Q_{xx} + lR_1(x,t)|Q|^2Q + V(x,t)Q = 0, \tag{1.2}$$

which relates to Eq.(1) of [9]. Let $l = 0$, n, k is still arbitrary constants, (1.1) collapses to the power law nonlinear Schrödinger equations with variable coefficients

$$\mathrm{i}Q_t + D(x,t)Q_{xx} + kR_2(x,t)|Q|^{2n}Q + V(x,t)Q = 0, \tag{1.3}$$

which relates to Eq.(1) of [19] and Eq.(2) of [24]. Let $n = 2$, l, k are still arbitrary constants, (1.1) collapses to the parabolic law or CQ law nonlinear Schrödinger equations with variable coefficients

$$\mathrm{i}Q_t + D(x,t)Q_{xx} + (lR_1(x,t)|Q|^2 + kR_2(x,t)|Q|^4)Q + V(x,t)Q = 0, \tag{1.4}$$

which relates to Eq.(1) of [11] with $\gamma(x,t) = 0$. If $V(x,t) = 0$, $D(x,t)$,$R_1(x,t)$,$R_2(x,t)$ are constant coefficients, (1.1) become an autonomous DPNLS equations, which describes the saturation of the nonlinear refractive index, and also serves as a basic model to describe the solitons in photovoltaic-photorefractive materials such as LiNbO3 [2]. However, if $V(x,t), D(x,t), R_1(x,t), R_2(x,t)$ are the functions of x and t respectively, we can't obtain solutions by using the traveling wave method of [27]. To deal with this problem, we hope to use the similarity transformation to obtain solitary waves of (1.1).

In recent years, the similarity transformation has been used to obtain the solution of the nonlinear Schrödinger equations. In [9, 11], the authors solve the nonlinear Schrödinger equations with variable coefficients by using the similarity transformation. In [6–8], the authors also skillfully obtain solitary wave for the coupled nonlinear Schrödinger equations with variable coefficients by using the similarity transformation. In our paper, we firstly deduce the generalized nonautonomous DPNLS to the usual autonomous DPNLS equation (2.2) and obtain stationary solitary waves of the usual autonomous DPNLS (2.2), then using Galilean invariance, we can obtain moving solitary waves of (2.2). Finally using similarity transformation, we could obtain solitary waves of (1.1), so we only need to study solitary waves of (2.2).

Equation (1) can be viewed as the evolution equation $Q_t = \frac{\delta\mathcal{H}}{\delta(\mathrm{i}Q^*)}$, where $\mathcal{H}$ is the Hamiltonian function

$$\mathcal{H} = -\int_{-\infty}^{+\infty} [Q^* D(x,t)Q_{xx} + lR_1(x,t)|Q|^{n+2} + kR_2(x,t)|Q|^{2n+2} + V(x,t)|Q|^2]dx.$$

Generally speaking, the Hamiltonian $\mathcal{H}$ in our model (1) is not conserved. It will be seen that this situation would be changed by employing a similarity transformation technique, and the Hamiltonian $\mathcal{H}$ can only be conserved under some special cases.

The organization of this paper is as follows: In Section 2, we map the generalized nonautonomous DPNLS into the DPNLS by using the similarity transformation, then through some computation, we get stationary solitary waves of the nonautonomous DPNLS. In Section 3, we design two interesting external potentials such as space-quadratic potential and optical super-lattice potential with DP nonlinearity, which have strong physical meanings in the nonlinear science. Moreover, we obtain solitary waves for these two interesting nonautonomous DPNLS with different values of k, l, n.

2. Solitary waves of nonautonomous NLSE

2.1. The similarity transformation

In this section, the generalized nonautonomous DPNLS (1.1) will be reduced into the autonomous DPNLS by the similarity transformation. In other words, solitary waves of the generalized nonautonomous DPNLS can be written as

$$Q(x,t) = q(X,T)p(x,t)e^{\mathrm{i}\phi(x,t)}, \tag{2.1}$$

where $X = X(x,t)$, $T = T(t)$, q, p, ϕ are real smooth functions of x and t. Using (2.1), we can reduce the generalized nonautonomous DPNLS (1.1) into the usual autonomous DPNLS

$$\mathrm{i}q_T + q_{XX} + (l|q|^n + k|q|^{2n})q = 0, \tag{2.2}$$

where $q(X,T)$ is solution of (2.2), $l|q|^n + k|q|^{2n}$ is dual-power nonlinearity. In the following computation process, if T is only expressed by t, we can eliminate q_{XT} and obtain (2.2) by setting different function D, R_1, R_2, V. Take (2.1) into (1.1), we obtain

$$\begin{aligned}&\mathrm{i}e^{\mathrm{i}\phi}(pq_X X_t + pq_T T_t + p_t q + \mathrm{i}pq\phi_t) + De^{\mathrm{i}\phi}(2p_x q_X X_x + 2\mathrm{i}pq_X X_x\phi_x + pq_{XX}X_x^2\\&+ pq_X X_{xx} + 2\mathrm{i}p_x q\phi_x + p_{xx}q - pq\phi_x^2 + \mathrm{i}pq\phi_{xx}) + (lR_1|pqe^{\mathrm{i}\phi}|^n + kR_2|pqe^{\mathrm{i}\phi}|^{2n})pqe^{\mathrm{i}\phi}\\&+ Vqpe^{\mathrm{i}\phi} = 0,\end{aligned} \tag{2.3}$$

where $D = D(x,t), R_1 = R_1(x,t), R_2 = R_2(x,t), \phi = \phi(x,t)$ are arbitrary functions of x and t. In order to mapping (1.1) to (2.2), let

$$D = \frac{T_t}{2{X_x}^2}, \quad R_1 = \frac{T_t}{p^n}, \quad R_2 = \frac{T_t}{p^{2n}}, \tag{2.4}$$

then take (2.4) into (2.3), we obtain

$$\begin{aligned}&\mathrm{i}q_T + q_{XX} + (l|q|^n + k|q|^{2n})q + (\frac{\mathrm{i}q_X X_t}{T_t} + \frac{\mathrm{i}p_t q}{pT_t} - \frac{q\phi_t}{T_t}) + (\frac{p_x q_X}{pX_x} + \frac{\mathrm{i}q_X\phi_x}{X_x} + \frac{q_X X_{xx}}{2X_x^2}\\&\quad + \frac{\mathrm{i}p_x q\phi_x}{pX_x^2} + \frac{p_{xx}q}{2pX_x^2} - \frac{q\phi_x^2}{2X_x^2} + \frac{\mathrm{i}q\phi_{xx}}{2X_x^2}) + \frac{Vq}{T_t} = 0,\end{aligned} \tag{2.5}$$

or they also can be written as

$$\mathrm{i}q_T + q_{XX} + (l|q|^n + k|q|^{2n})q + \frac{(\mathrm{i}k_1 + k_2)q_X}{2p{X_x}^2 T_t} + \frac{(\mathrm{i}k_3 + k_4)q}{2pX_x^2 T_t} = 0, \tag{2.6}$$

where k_i, $i = 1,2,3,4$ are given by

$$k_1 = 2pX_x^2 X_t + 2pX_x\phi_x T_t, \tag{2.7}$$

$$k_2 = 2p_x X_x T_t + pX_{xx} T_t, \tag{2.8}$$

$$k_3 = 2p_t X_x^2 + 2p_x \phi_x T_t + p\phi_{xx} T_t, \tag{2.9}$$

$$k_4 = p_{xx} T_t - 2pX_x^2 \phi_t - p\phi_x^2 T_t + 2VpX_x^2. \tag{2.10}$$

(2.6) is very close to (2.2), so we need to do an assumption $k_1 = k_2 = k_3 = k_4 = 0$, which is vital step to our transformation. From $k_2 = 0$, we obtain $p^2 X_x = f_1^2(t)$. From $k_1 = 0$, we obtain $\phi_x = \frac{-X_x X_t}{T_t}$. From $k_3 = 0$, we obtain $\frac{p^2}{X_x} = \frac{1}{F^2(x)}$, where $F(x)$ is integral function about x. Furthermore, from the assumption, we obtain the expression of p, X, ψ

$$p = \sqrt{\frac{f_1(t)}{F(x)}}, \quad X = \int F(x) f_1(t) dx + f_2(t), \tag{2.11}$$

$$\phi = \int \frac{(\int F(x) f_{1t} dx + f_{3t}) F(x) f_1(t)}{T_t} dx + f_3(t), \tag{2.12}$$

where $f_1(t), f_2(t), f_3(t)$ are integral function of t. Moreover, take the expression of X, p, T in (2.11)-(2.12) into (2.4), we obtain

$$D = \frac{T_t}{2F^2 f_1(t)^2} \quad R_1 = T_t \left(\frac{F}{f_1(t)}\right)^{\frac{n}{2}}, \quad R_2 = T_t \left(\frac{F}{f_1(t)}\right)^{\frac{n}{2}}. \tag{2.13}$$

Now the expression of the external potential $V(x,t)$ is given by $k_4 = 0$

$$V = \frac{8f_1^2(t)\phi_t F^4(x) + 4T_t \phi_x^2 F^2(x) - 3T_t F_x^2 + 2T_t F F_{xx}}{8F^4(x) f_1^2(t)}. \tag{2.14}$$

From the equation (2.11)-(2.12), we find that the external potential $V(x,t)$ isn't arbitrary function but relates to X, p, T, ϕ which are decided by $F(x)$, $f_1(t)$, $f_2(t)$, $f_3(t)$, T.

Thus similarity transformations (2.1) indeed map the generalized nonautonomous DPNLS to the nautonomous DPNLS systems, we only need to obtain solution of the nautonomous DPNLS equation (2.2) in the next section. Equations (2.11)-(2.12) and (2.14) can be seen as integrability condition on the generalized nonautonomous DPNLS (1.1). External potential (2.14) and DP nonlinearity (2.13) have different expression by choices of arbitrary functions $F(x), f_1(t)(F(x)f_1(t) > 0)$, $f_2(t), f_3(t), T(t)$.

2.2. Solitary waves of the autonomous DPNLS

In this section, we get the stationary solitary waves of the autonomous DPNLS. Using Galilean invariance, we obtain moving solitary waves of (2.2). The form of the stationary solitary waves of (2.2) can be written as

$$q(X,T) = u(X)e^{\mathrm{i}\mu T}, \tag{2.15}$$

where μ is a positive propagation constant. $u(X)$ obeys the nonlinear ordinary differential equation (ODE)

$$u_{XX} - \mu u + (lu^n + ku^{2n})u = 0, \tag{2.16}$$

where $u(X)$ is a localized real function. Multiplying (2.16) by u_X and integrating once, after simple computation, we obtain the explicit expression of u as

$$u(X) = \left[\frac{A}{B+\cos h(CX)}\right]^{\frac{1}{n}}, \tag{2.17}$$

$$A = \frac{(2+n)\mu B}{l},\ B = \mathrm{sign}(l)\left[1+\frac{(n+2)^2 k\mu}{(n+1)l^2}\right]^{-\frac{1}{2}},\ C = n\sqrt{\mu},$$

where $(4\mu k + l^2) > 0$ is the condition of existence of solitary wave [27]. Moreover, we can obtain precisely the soliton solution of equation (1.1) from the soliton solution of (2.17) and the transformation (2.1) and (2.15). The soliton solution of equation (1.1) is

$$Q(x,t) = \left[\frac{A}{B+\cos h(CX)}\right]^{\frac{1}{n}} p(x,t)e^{\mathrm{i}(\mu T+\phi(x,t))}. \tag{2.18}$$

In order to better illustrate the property of dynamical evolution of solution (2.18) of (1.1), we set different functions $F(x), f_1(t), f_2(t), f_3(t), T(t)$, simultaneously $T_1 = 0, T_2 = 0, T_3 = 0, c_0 = 1$ and then obtain expressions of D, R_1, R_2, V, which must satisfy our condition (2.11)-(2.12) and (2.14).

3. Designed external potentials

In this section, we will show two interesting nonautonomous DPNLS, where the potential functions V in equations (1.1) are specially chosen by setting different functions $F(x), f_1(t),\ f_2(t),\ f_3(t),\ T(t)$. In the following, we mainly introduce nonautonomous DPNLS with space-quadratic potential and optical super-lattice potential, simultaneously, the corresponding (DP) nonlinearity have significant expressions, which have important physical applicants in nonlinear optics and BEC problems [6,8,9,11].

3.1. Space-quadratic potential

Firstly, from equations (2.11)-(2.14), we set $F = F_0,\ f_1 = a_0\,\mathrm{sech}(t),\ f_2 = 1,\ f_3 = 1$ where $F_0 a_0 > 0$. Then, the X, p, ϕ, T are designed as

$$X = F_0 a_0\,\mathrm{sech}(t)x + 1, \quad T = a_1\tanh(t), \tag{3.1}$$

$$p = \sqrt{\frac{a_0}{F_0}\mathrm{sech}(t)}, \quad \phi = -\frac{a_0^2F_0^2}{2a_1}\tanh(t)x^2 + 1, \tag{3.2}$$

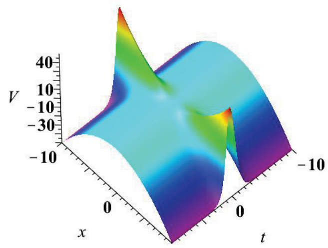
where a_1 is constant. From (3.1)-(3.2), we obtain nonautonomous DPNLS

$$\mathrm{i}Q_t + D(t)Q_{xx} + (lR_1(t)|Q|^n + kR_2(t)|Q|^{2n})Q + V(x,t)Q = 0, \tag{3.3}$$

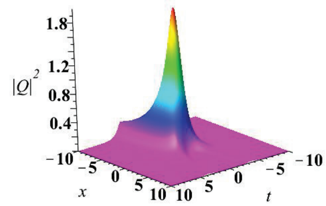
where

$$D(t) = \frac{a_1}{F_0^2a_0^2}, \qquad R_1(t) = a_1\mathrm{sech}^2(t)\left(\frac{a_0}{F_0\cosh(t)}\right)^{-\frac{n}{2}}, \tag{3.4}$$

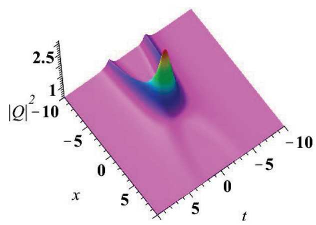
$$R_2(t) = a_1\,\mathrm{sech}(t)^2\left(\frac{a_0}{(F_0\cosh(t))}\right)^{-n}, \qquad V(x,t) = F_0^2a_0^3\frac{2\,\mathrm{sech}^2(t)-1}{2a_1}x^2, \tag{3.5}$$



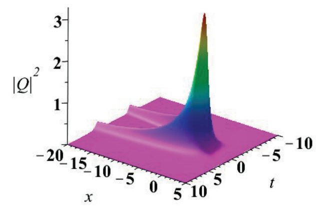
(a) V(x,t)



(b) $l = 1, k = 0, n = 2$



(c) $l = -1, k = 1, n = 2$



(d) $l = -1, k = 1, n = \frac{3}{2}$

Figure 1. Solitary waves of the nonautonomous DPNLS.(a), Space-quadratic potential. (b), solitary waves (3.6) with $l = 1, k = 0, n = 2$ and $a_0 = a_1 = F_0 = 1$. (b), solitary waves (3.6) with $l = -1, k = 1, n = 2$ and $a_0 = a_1 = F_0 = 1$. (d), solitary waves (3.6) with $l = -1, k = 1, n = \frac{3}{2}$ and $a_0 = a_1 = F_0 = 1$.

where D, R_1, R_2 are functions only of t. V is space-quadratic potential which is plotted in Fig.1–(a). The space-quadratic potential has been studied in [11] and is significant in the (BECs) and nonlinear optics [3]. Equation (1.1) is the generalized dual-power nonlinear Schrödinger equations DPNLS with time coefficients. Solitary waves (2.1) can be written as

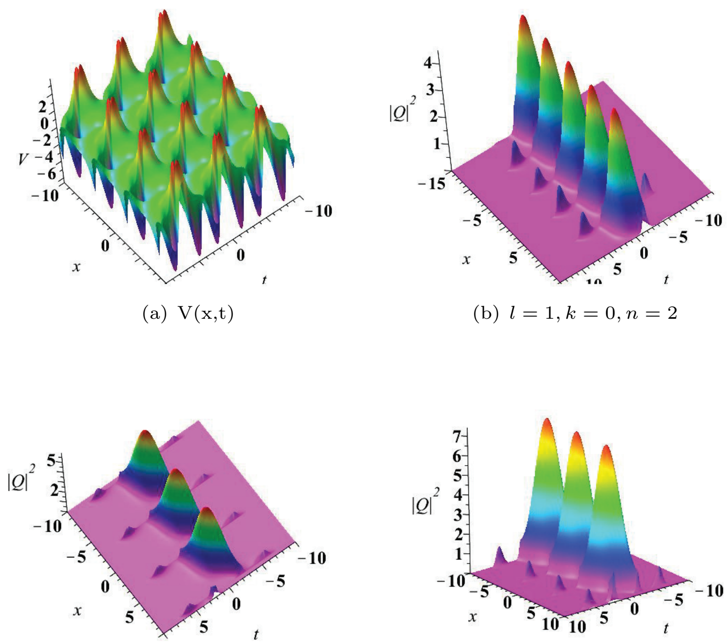
$$Q(x,t) = \frac{A\sqrt{a_0 \operatorname{sech}(t)}}{\sqrt{F_0}(B + \cosh(C(F_0 a_0 \operatorname{sech}(t)x + 1)))} e^{\mathrm{i}(\mu a_1 \tanh(t) - \frac{a_0^2 F_0^2}{2a_1} \tanh(t)x^2 + 1)}. \tag{3.6}$$

Profiles of $|Q|^2$ are plotted in Fig.1(b)-(d) with different l, n, k. Meanwhile, from (3.5), the amplitude of solitary waves is $\frac{A\sqrt{a_0 \operatorname{sech}(t)}}{\sqrt{F_0}(B+1)}$, depending on the changes of the parameter $f_1(t)$. Therefore, we obtain solitary wave by choosing different l, n, k, a_0, a_1.

3.2. Optical super-lattice potential

Firstly, from equations (2.11)-(2.14), we set $F = \frac{1}{b_0 \cos \omega_0 x + 1}$, $f_1 = 1 + b_1 \cos \omega_1 t$, $f_2 = t$, $f_3 = t$, where $b_0 > 1$, $b_1 > 0$, $\omega_i \in \mathbb{R}$, $i = 0, 1$. Then, p, X, ϕ, T are designed as

$$X = \frac{2b_1 \cos(\omega_1 t) + 2}{\omega_0 \sqrt{b_0^2 - 1}} \operatorname{arctanh}(\frac{b_0 - 1}{\sqrt{b_0^2 - 1}} \tan(\frac{\omega_0}{2} x)) + t, \quad T = t, \tag{3.7}$$



(a) V(x,t) (b) $l = 1, k = 0, n = 2$

(c) $l = -1, k = 1, n = 2$ (d) $l = -1, k = 1, n = \frac{3}{2}$

Figure 2. Solitary waves of the nonautonomous DPNLS. (a), Fourier-synthesized lattice potential.(b), solitary waves (3.14) with $l = 1, k = 0, n = 2$ and $b_0 = b_1 = \frac{1}{2}, \omega_0 = \omega_1 = 1$. (c), solitary waves (3.14) with $l = -1, k = 1, n = 2$ and $b_0 = b_1 = \frac{1}{2}, \omega_0 = \omega_1 = 1$. (d), solitary waves (3.14) with $l = -1, k = 1, n = \frac{3}{2}$ and $b_0 = b_1 = \frac{1}{2}, \omega_0 = \omega_1 = 1$.

$$p = \sqrt{(1 + b_1 \cos(\omega_1 t))(b_0 \cos(\omega_0 x) + 1)}, \tag{3.8}$$

$$\phi = \frac{1}{\omega_0 \sqrt{b_0^2 - 1}} \int [\frac{1}{b_0 \cos(\omega_0 x) + 1}(-2b_1\omega_1 \sin(\omega_1 t) \operatorname{arctanh}(\frac{b_0 - 1}{\sqrt{b_0^2 - 1}} \tan(\frac{\omega_0}{2}x)) + \omega_0 \sqrt{b_0^2 - 1})(\cos(\omega_1 t)b_1 + 1)]dx + t. \tag{3.9}$$

The variable coefficients in (1.1) are designed as

$$D = \left(\frac{b_0 \cos(\omega_0 x) + 1}{b_1 \cos(\omega_1 t) + 1}\right)^2, \tag{3.10}$$

$$R_1 = (1 + b_1 \cos(\omega_1 t))(b_0 \cos(\omega_0 x) + 1)^{-\frac{n}{2}}, \tag{3.11}$$

$$R_2 = (1 + b_1 cos(\omega_1 t))(b_0 \cos(\omega_0 x) + 1)^{-n}, \tag{3.12}$$

$$V = \phi_t + \frac{1}{2}\phi_x^2 - \frac{b_0^2 \omega_0^2 \sin^2(\omega_0)}{8(1 + b_1 \cos \omega_1 t)^2} + \frac{2b_0\omega_0^2 \cos(\omega_0 x)}{8(1 + b_1 \cos \omega_1 t)}, \tag{3.13}$$

where D, R_1, R_2, V are functions about space x and time t, V is fourier-synthesized lattice potential which is plotted in Fig.2–(a). The Fourier-synthesized lattice potential resembles a periodic sequence of hills in the transverse and longitudinal directions. In [10, 11, 21, 29], the authors consider the Fourier-synthesized lattice potential which have important physics meanings. In our paper, we consider the more widely Fourier-synthesized lattice potential. (1.1) with the Fourier-synthesized lattice potential (3.13) display nonuniform distribution both transverse and longitudinal directions in the nonlinear science.

Solitary waves (2.1) can be written as

$$Q(x,t) = \frac{A\sqrt{(1+b_1\cos(\omega_1 t))(b_0\cos(\omega_0 x)+1)}}{B+\cosh(C(2\frac{b_1\cos(\omega_1 t)+1}{\omega_0\sqrt{b_0^2-1}}\operatorname{arctanh}(\sqrt{\frac{b_0-1}{b_0+1}}\tan(\frac{\omega_0}{2}x))+t))} e^{\mathrm{i}(\mu t+\phi)}. \tag{3.14}$$

Profiles of $|Q|^2$ are plotted in Fig.2(b)-(d) with different l, n, k. Meanwhile, from (3.2), the amplitude of solitary waves is $\frac{A\sqrt{(1+b_1\cos(\omega_1 t))(b_0\cos(\omega_0 x)+1)}}{B+1}$, depending on the parameters $f_1(t), F(x)$. Therefore, the amplitude will be affected by all the parameters $l, n, k, b_0, b_1, \omega_0, \omega_2$, which leads to the complex changes of the solitary wave.

4. Conclusion

In this paper, we analytically obtain the solitary waves for the generalized dual-power nonlinear Schrödinger equations (DPNLS) with variable coefficients by using the similarity transformation. We design two interesting external potential and DP nonlinearity, and obtain solitary waves of corresponding nonautonomous DPNLS, which have important physical meanings in the nonlinear science. The method we used in this paper can be extended to obtain solitary waves of the higher-dimensional generalized DPNLS and study their interaction properties. Stability of solitary waves for (1.1) with time coefficients will be further discussed in a separate paper. In addition, Based on some novel physical phenomena in nonlinear optics and the BECs, we predict that new exact solutions of the generalized DPNLS will exist, such as rogue waves.

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